

## Equilibrium of a Rigid Body

In this chapter the fundamental concepts of rigid-body equilibrium will be discussed. It will be shown that equilibrium requires both a *balance of forces*, to prevent the body from translating with accelerated motion, and a *balance of moments*, to prevent the body from rotating.

Many types of engineering problems involve symmetric loadings and can be solved by projecting all the forces acting on a body onto a single plane. Hence, in the first part of this chapter, the equilibrium of a body subjected to a *coplanar* or *two-dimensional force system* will be considered. Ordinarily the geometry of such problems is not very complex, so a scalar solution is suitable for analysis. The more general discussion of rigid bodies subjected to *three-dimensional force systems* is given in the second part of this chapter. It will be seen that many of these types of problems can best be solved by using vector analysis.

### 4.1 Conditions for Rigid-Body Equilibrium

The *equations of equilibrium* for a rigid body can be summarized as follows:

$$\begin{aligned}\sum F &= 0 \\ \sum M_O &= 0\end{aligned}\quad (4-1)$$

These equations require that a rigid body will be in equilibrium provided the sum of all the external forces acting on the body is equal to zero and the sum of the moments of the external forces about a point is equal to zero. The fact that these conditions are *necessary* for equilibrium has now been proven. They are also *sufficient* conditions. To show this, let us assume that the body is *not* in equilibrium, and yet the force system acting on it satisfies Eqs. 4-1. Suppose that an *additional force*  $F'$  is required to hold the body in equilibrium. As a result, the equilibrium equations become

$$\begin{aligned}\sum F + F' &= 0 \\ \sum M_O + M'_O &= 0\end{aligned}$$

where  $M'_O$  is the moment of  $F'$  about  $O$ . Since  $\sum F = 0$  and  $\sum M_O = 0$ , then we require  $F' = 0$  (also  $M'_O = 0$ ). Consequently, the additional force  $F'$  is not required for holding the body, and indeed Eqs. 4-1 are *also sufficient conditions*.

### Equilibrium in Two Dimensions

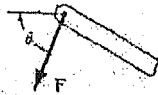
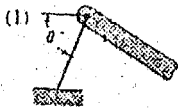
#### 4.2 Free-Body Diagrams

Successful application of the equations of equilibrium requires a complete specification of *all* the known and unknown forces that act *on* the body. The best way to account for these forces is to draw the body's free-body diagram. This diagram is a sketch of the outlined shape of the

body which represents it as being *isolated* or "free" from its surroundings. On this sketch one then shows *all* the forces (or their resultants) which the surroundings exert *on the body*. Obviously, if the free-body diagram is correctly drawn, the effects of all the applied forces and couples acting on the body can be accounted for when the equations of equilibrium are applied. For this reason, *a thorough understanding of how to draw a free-body diagram is of primary importance for solving problems in mechanics*. It is the preliminary step before applying the equations of equilibrium

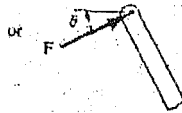
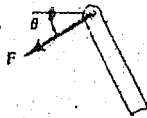
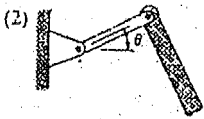
Table 4-1 Supports for Rigid Bodies Subjected to Two-Dimensional Force System

Types of Connection      Reaction



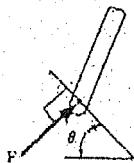
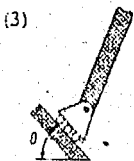
Cable

One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.



weightless link

One unknown. The reaction is a force which acts along the axis of the link.



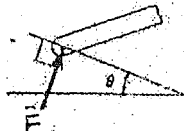
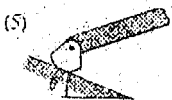
roller

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.



roller or pin in confined smooth slot

One unknown. The reaction is a force which acts perpendicular to the slot.

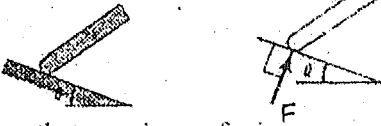


rocker

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Types of Connection      Reaction ( Table 4-1 Contd )

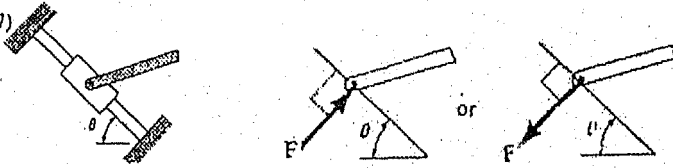
(6)



smooth contacting surface

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(7)



member pin connected to collar on smooth rod

One unknown. The reaction is a force which acts perpendicular to the rod.

(8)



smooth pin or hinge

Two unknowns. The reactions are two components of force, or the magnitude and direction  $\phi$  of the resultant force. Note that  $\phi$  and  $\theta$  are not necessarily equal (usually not, unless the rod shown is a link as in (2)).

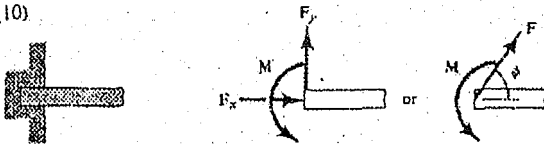
(9)



member fixed connected to collar on smooth rod

Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.

(10)



fixed support

Three unknowns. The reactions are the couple and the two force components, or the couple and the magnitude and direction  $\phi$  of the resultant force.

## PROCEDURE FOR DRAWING A FREE-BODY DIAGRAM

To construct a free-body diagram for a rigid body or group of bodies considered as a single system, the following steps should be performed:

*Step 1.* Imagine the body to be *isolated* or 'cut' 'free' from its constraints and connections, and

draw (sketch) its outlined shape.

**Step 2.** Identify all the external forces and couples that act on the body. Forces and couples generally encountered are those due to (1) applied loadings, (2) reactions occurring at the supports or at points of contact with other bodies (see Table 4-1), and (3) the weight of the body. To account for all these forces, it may help to trace over the boundary, carefully noting each force or couple acting on it.

**Step 3.** Indicate the dimensions of the body necessary for computing the moments of forces. The forces and couples that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and direction angles of forces and couples that are *unknown*. In particular, if a force or couple has a known line of action but unknown magnitude, the arrowhead which defines the sense of the vector can be assumed. The correctness of the assumed sense will become apparent after solving the equilibrium equations for the unknown magnitude. By definition, the *magnitude* of a vector is *always positive*, so that if the solution yields a "negative" scalar, the *minus sign* indicates that the vector's sense is *opposite* to that which was originally assumed.

#### Example 4-1

Two smooth balls A and B, each having a mass of 2 kg, rest between the inclined planes shown in Fig. 4-1a. Draw the free-body diagrams for ball A, ball B, and balls A and B together.

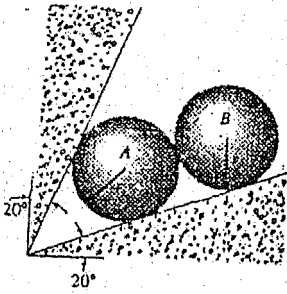
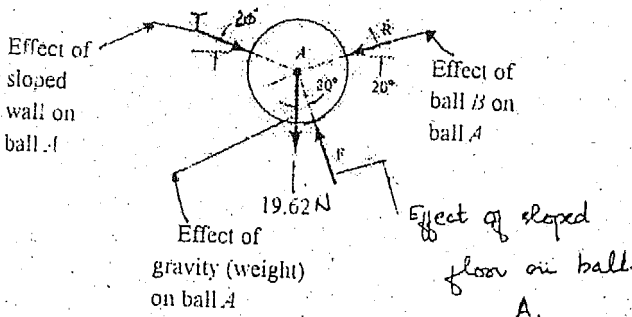


Fig. 4-1a

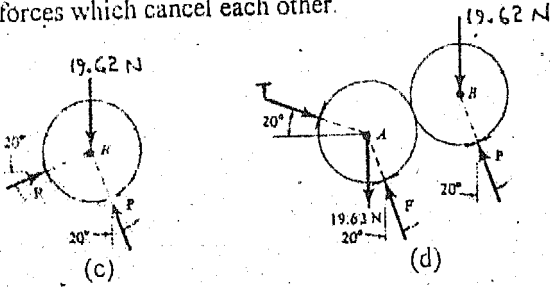
(a)

#### SOLUTION

The free-body diagram for ball A is shown in Fig. 4-1b. Note that the weight of the ball is calculated as  $W = 2(9.81) \approx 19.62 \text{ N}$ . Since all contacting surfaces are *smooth*, the reactive forces T, F, R act in a direction *normal* to the tangent at their surfaces of contact.

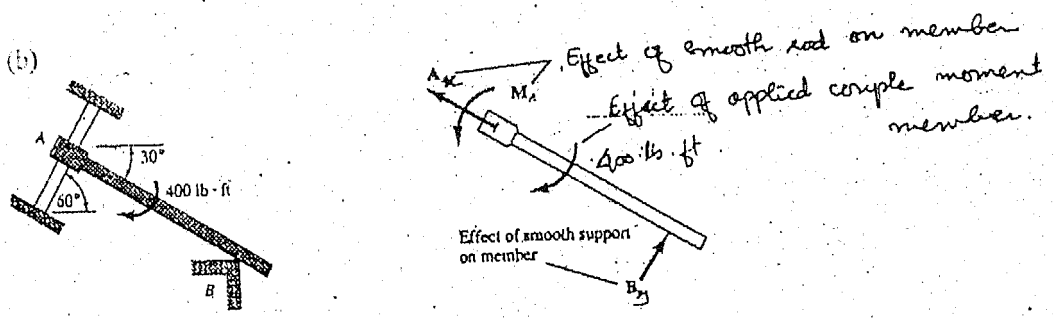
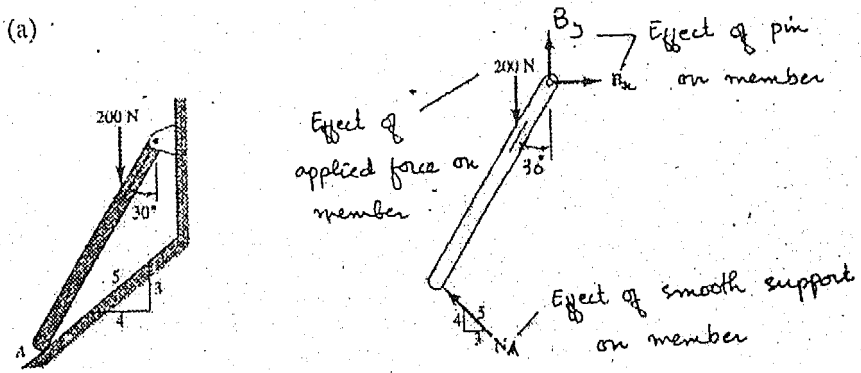


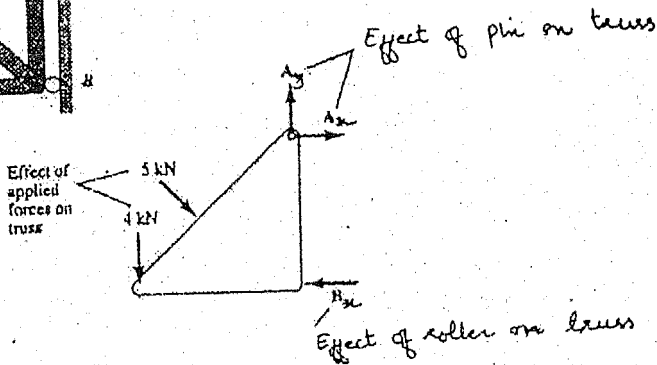
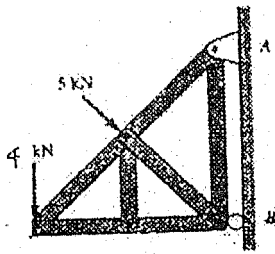
The free-body diagram of ball *B* is shown in Fig. 4-1c. In particular, note that *R*, representing the force of ball *A* on ball *B*, Fig. 4-1c, is equal and opposite to *R* representing the force of ball *B* on ball *A*, Fig. 4-1b. This is a consequence of Newton's third law of motion. The free-body diagram of both balls combined ("system") is shown in Fig 4-1d. The contact force *R*, which acts between *A* and *B* is considered as an *internal* force and hence is not shown on the free-body diagram. That is, it represents a pair of equal but opposite collinear forces which cancel each other.



Example 4-2

The free-body diagram of each object in Fig. 4-2 is drawn and the forces acting on the object are identified. The weight of the objects is neglected except where indicated.





### 4.3 Equations of Equilibrium

In Sec. 4.1 we developed the two equations which are both necessary and sufficient for the equilibrium of a rigid body, namely,  $\sum F = 0$  and  $\sum M_O = 0$ . When the body is subjected to a system of forces, which all lie in the  $x$ - $y$  plane, then the forces can be resolved into their  $x$  and  $y$  components. Consequently, the conditions for equilibrium in two dimensions are

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_O &= 0 \end{aligned} \tag{4-2}$$

Here  $\sum F_x$  and  $\sum F_y$  represent, respectively, the algebraic sums of the  $x$  and  $y$  components of all the forces acting on the body, and  $\sum M_O$  represents the algebraic sum of the moments of all these force components about an axis perpendicular to the  $x$ - $y$  plane and passing through the arbitrary point  $O$ , which may lie either on or off the body

#### Example 4-3

The beam shown in Fig. 4-3a is pin-connected at A and rests against a roller at B. Compute the horizontal and vertical components of reaction at the pin A.

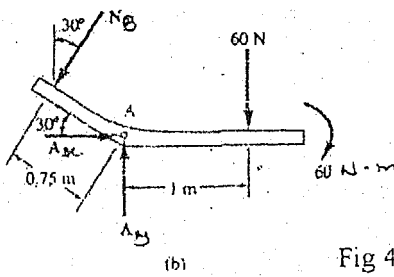
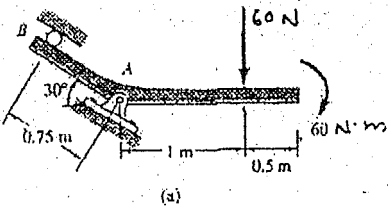


Fig 4-3

**SOLUTION**

**Free-Body Diagram.** As shown in Fig. 4-3b, the reaction  $N_B$  is perpendicular to the beam at  $B$ , since the support is a roller. Also, horizontal and vertical components of reaction are represented at  $A$ , even though the base of the pin support is tilted.

**Equations of Equilibrium.** Summing moments about  $A$ , we obtain a direct solution for  $N_B$

$$\zeta + \sum M_A = 0; \quad -60 \text{ N} \cdot \text{m} - 60 \text{ N} (1 \text{ m}) + N_B (0.75 \text{ m}) = 0$$

$$N_B = 160 \text{ N}$$

Using this result,

$$\rightarrow + \sum F_x = 0; \quad A_x - 160 \sin 30^\circ \text{ N} = 0$$

$$A_x = 80.0 \text{ N}$$

$$+ \uparrow \sum F_y = 0; \quad A_y - 160 \cos 30^\circ \text{ N} - 60 \text{ N} = 0$$

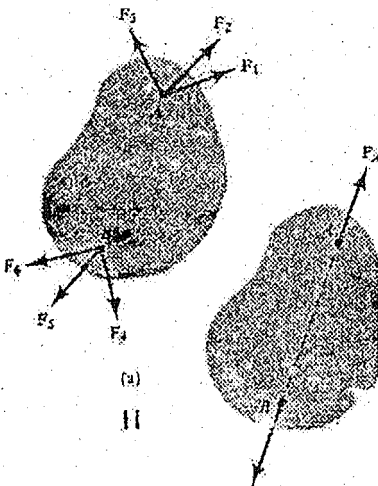
$$A_y = 199 \text{ N} \quad \text{Ans.}$$

**4-4 Two and Three Force Members**

The solution to some equilibrium problems can be simplified if one is able to recognize members that are subjected to only two or three forces.

**Two-Force Members**

When a member is subjected to *no couples* and forces are applied at only two points on a member, the member is called a *two-force member*. An example of this situation is shown in Fig. 4-4a. The forces at  $A$  and  $B$  are first summed to obtain their respective *resultants*  $F_A$  and  $F_B$ , Fig. 4-4b. These two forces will maintain *translational or force equilibrium* provided  $F_A$  is of equal magnitude and opposite direction to  $F_B$ . Furthermore, *rotational or moment equilibrium* is satisfied if  $F_A$  is *collinear* with  $F_B$ . As a result, the line of action of both forces is known, since it always passes through  $A$  and  $B$ . Hence, only the force magnitude must be determined or stated.



(b) Fig 4-4

### Three-Force Members

If a member is subjected to three coplanar forces, then it is necessary that the forces be either concurrent or parallel if the member is to be in equilibrium. To show this, consider the body in Fig. 4-5 and suppose that any two of the three forces acting on the body have lines of action that intersect at point  $O$ . To satisfy moment equilibrium about  $O$ , i.e.,  $\sum M_O = 0$ , the third force must also pass through  $O$ , which then makes the force system *concurrent*. If two of the three forces are parallel, the point of concurrency,  $O$ , is considered to be at "infinity" and the third force must be parallel to the other two forces to intersect at this "point." Since the three forces are concurrent at a point, only the force equilibrium equations ( $\sum F_x = 0$ ,  $\sum F_y = 0$ ) must be satisfied.

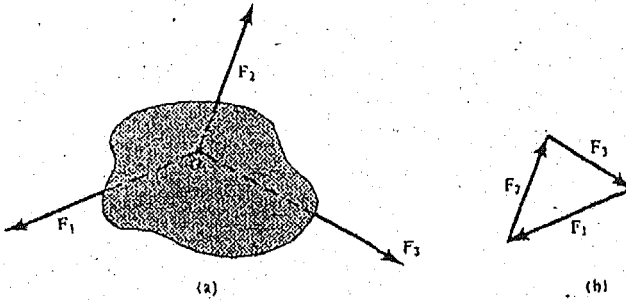


Fig 4-5

### Example 4-4

The lever  $ABC$  is pin-supported at  $A$  and connected to a short link  $BD$  as shown in Fig. 4-6a. If the weight of the members is negligible, determine the force developed on the lever at  $A$ .

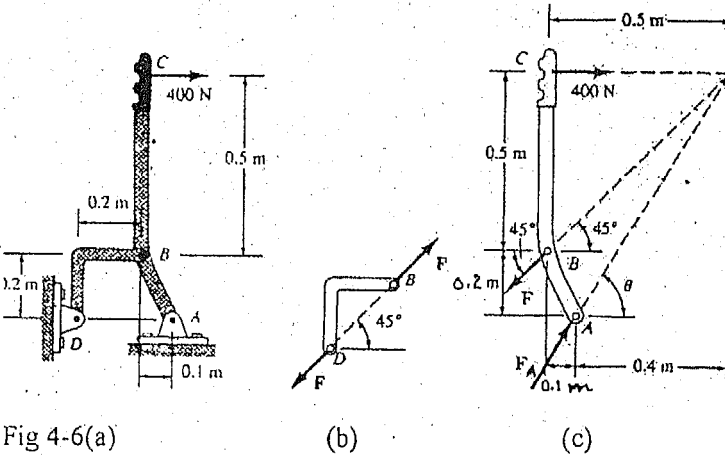


Fig 4-6(a)

(b)

(c)

### SOLUTION

**Free-Body Diagrams.** As shown by the free-body diagram, Fig. 4-6 b, the short link  $BD$  is a *two-force member*, so the *resultant forces* at pins  $D$  and  $B$  must be equal, opposite, and collinear. Although the magnitude of the force is unknown, the line of action is known, since it passes through  $B$  and  $D$ .

Lever  $ABC$  is a *three-force member*, and therefore the three nonparallel forces acting on it

must be concurrent at  $O$ , Fig. 4-6c. In particular, note that the force  $F$  on the lever is equal but opposite to  $F$  acting at  $B$  on the link. Why? The distance  $CO$  must be 0.5 m, since the lines of action of  $F$  and the 400-N force are known.

**Equations of Equilibrium.** By requiring the force system to be concurrent at  $O$ , it is necessary that  $\sum M_O = 0$ . Hence, the angle  $\theta$  which defines the line of action of  $F_A$  can be determined from trigonometry,

$$\theta = \tan^{-1} \left( \frac{0.7}{0.4} \right) = 60.3^\circ \quad \text{Ans}$$

Applying the force equilibrium equations yields

$$\rightarrow \sum F_x = 0; \quad F_A \cos 60.3^\circ - F \cos 45^\circ + 400 \text{ N} = 0$$

$$+\uparrow \sum F_y = 0; \quad F_A \sin 60.3^\circ - F \sin 45^\circ = 0$$

Solving, we get

$$F_A = 1075 \text{ N} \quad \text{Ans.}$$

$$F = 1320 \text{ N}$$

We can also solve this problem by representing the force at  $A$  by its two components  $A_x$  and  $A_y$ , and applying  $\sum M_A = 0$ ,  $\sum F_x = 0$ ,  $\sum F_y = 0$ . Once  $A_x$  and  $A_y$  are determined, how would you find  $F_A$  and  $\theta$ ?

## Equilibrium in Three Dimensions

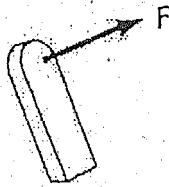
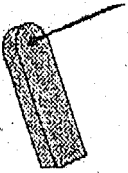
### 4-5 Free Body Diagrams

#### Table 4-2 Supports for Rigid Bodies Subjected to Three Dimensional Force Systems

Types of Connection

Reaction

(1)



cable

One unknown. The reaction is a force which acts away from the member in the direction of the cable.

(2)



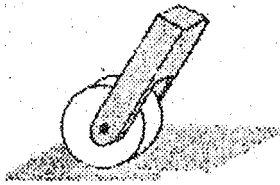
smooth surface support

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Types of Connection

Reaction Table 4-2( Contd )

(3)



on a smooth surface



One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

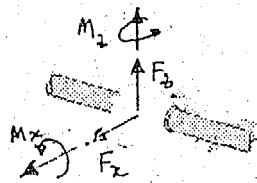
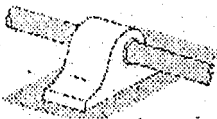
(4)



ball and socket

Three unknowns. The reactions are three rectangular force components.

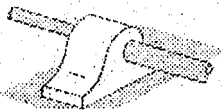
(5)



single journal bearing

Four unknowns. The reactions are two force, and two couple components which act perpendicular to the shaft.

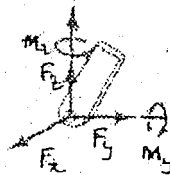
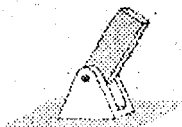
(6)



single thrust bearing

Five unknowns. The reactions are three force and two couple components

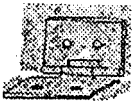
(7)



single smooth pin

Five unknowns. The reactions are three force and two couple components

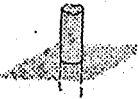
(8)



single hinge

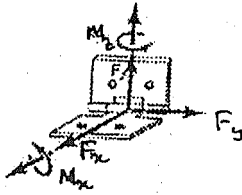
Five unknowns. The reactions are three force and two couple components.

(9)



fixed support

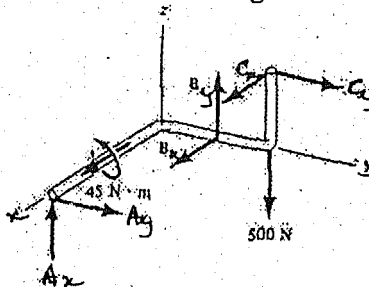
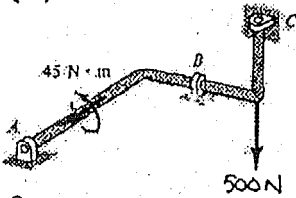
Six unknowns. The reactions are three force and three couple components.



Example 4-5

Several examples of objects along with their associated free-body diagrams are shown in Fig. 4-7. In all cases, the  $x, y, z$  axes are established and the unknown reaction components are indicated in the positive sense. The weight of the members is neglected

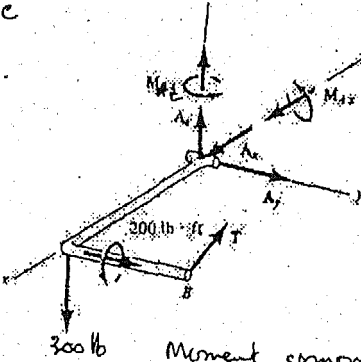
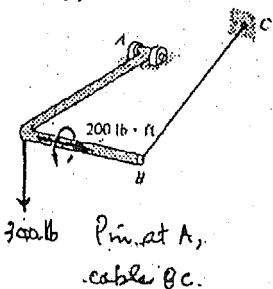
(a)



Proxely aligned journal bearings at A, B, C

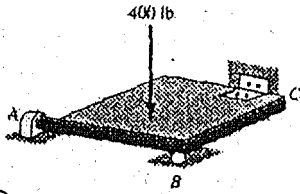
The force reactions at the bearings are sufficient for force and moment equilibrium since they prevent the shaft from rotating about each of the coordinate axes.

(b)

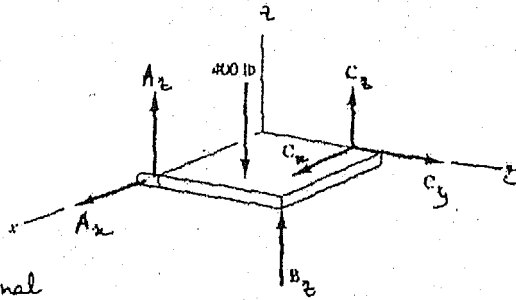


Moment components are developed by the pin on the rod in order to prevent rotating about the  $x$  and  $z$  axes

(c)



Properly aligned journal casing at A and hinge at C. Roller at B.



Only force reactions are developed at the bearing and hinge.

Fig. 4-7 (a-c)

#### 4-6 Equation of Equilibrium

The conditions for equilibrium of a rigid body subjected to a three-dimensional force system require that both the *resultant* force and *resultant* couple moment acting on the body be equal to zero.

#### Vector Equations of Equilibrium

The two conditions for equilibrium of a rigid body may be expressed mathematically in vector form as

$$\begin{aligned}\sum F &= 0 \\ \sum M_O &= 0\end{aligned}\quad (4-3)$$

where  $\sum F$  is the vector sum of all the external forces acting on the body and  $\sum M_O$  is the sum of the moments of all these forces about any point  $O$  located either on or off the body.

#### Scalar Equations of Equilibrium

If all the applied external forces and moments are expressed in Cartesian vector form and substituted into Eqs. 4-3, we have

$$\sum F = \sum F_x i + \sum F_y j + \sum F_z k = 0 \quad \sum M_O = \sum M_x i + \sum M_y j + \sum M_z k = 0$$

Since the  $i$ ,  $j$ , and  $k$  components are independent from one another, the above equations are satisfied provided

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0\end{aligned}\quad (4-4a)$$

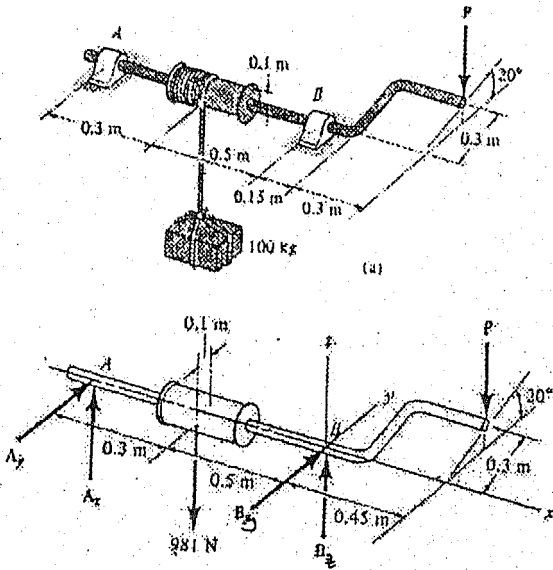
and

$$\begin{aligned}\sum M_x &= 0 \\ \sum M_y &= 0 \\ \sum M_z &= 0\end{aligned}\quad (4-4b)$$

These *six scalar equilibrium equations* may be used to solve for at most six unknowns shown on the free-body diagram. Equations 4-4a express the fact that the sum of the external force components acting in the  $x$ ,  $y$ , and  $z$  directions must be zero, and Eqs. 4-4b require the sum of the moment components about the  $x$ ,  $y$ , and  $z$  axes to be zero.

### Example 4-6

The windlass shown in Fig. 4-8a is supported by two smooth journal bearings  $A$  and  $B$  which are properly aligned on the shaft. Determine the magnitude of the vertical force  $P$  that must be applied to the handle to maintain equilibrium of the 100-kg crate. Also calculate the reactions at the bearings.



(b) Fig 4-8

### SOLUTION (SCALAR ANALYSIS)

**Free-Body Diagram.** Since the bearings at  $A$  and  $B$  are aligned correctly, *only* force reactions occur at these supports, Fig. 4-8b. Why are there no moment reactions?

**Equations of Equilibrium.** Summing moments about the  $x$  axis yields a direct solution for  $P$ . Why? For a scalar moment summation, it is necessary to compute the moment of each force as the product of the force magnitude and the *perpendicular distance* from the  $x$  axis to the line of action of the force. Using the right-hand rule and assuming positive moments act in the  $+i$  direction, we have

$$\sum M_x = 0; \quad 981 \text{ N}(0.1 \text{ m}) - P(0.3 \cos 20^\circ \text{ m}) = 0$$

$$P = 348.0 \text{ N} \quad \text{Ans.}$$

Using this result and summing moments about the  $y$  and  $z$  axes yields

$$\sum M_y = 0; \quad -981 \text{ N}(0.5 \text{ m}) + A_z(0.8 \text{ m}) - (348.0 \text{ N})(0.4 \text{ m}) = 0$$

$$A_z = 417.4 \text{ N} \quad \text{Ans.}$$

$$\sum M_z = 0; \quad -A_y(0.8 \text{ m}) = 0$$

$$A_y = 0 \quad \text{Ans.}$$

The reactions at  $B$  are obtained by a force summation, using the results computed above.

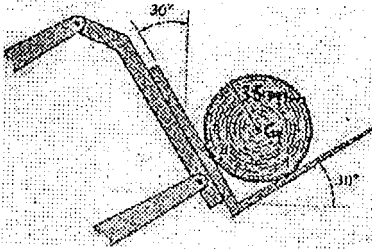
$$\sum F_y = 0; \quad 0 + B_y = 0 \quad B_y = 0 \quad \text{Ans.}$$

$$\sum F_z = 0; \quad 417.4981 + B_z - 348.0 = 0 \quad B_z = 911.6 \text{ N} \quad \text{Ans.}$$

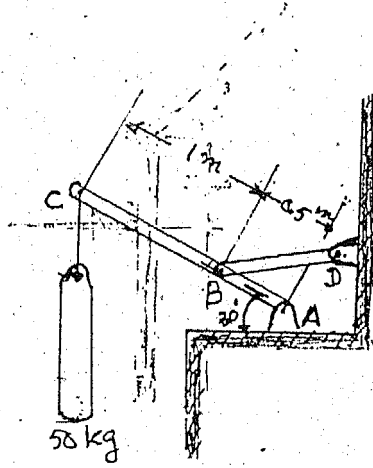
As shown on the free-body diagram, the supports do not provide resistance against translation in the x direction. Hence, the windlass is only partially constrained.

**Problems**

4-1 Draw the free-body diagram of the 50-kg roll of paper which has a center of mass at G and rests on the smooth blade of the paper hauler. Explain the significance of each force acting on the diagram.



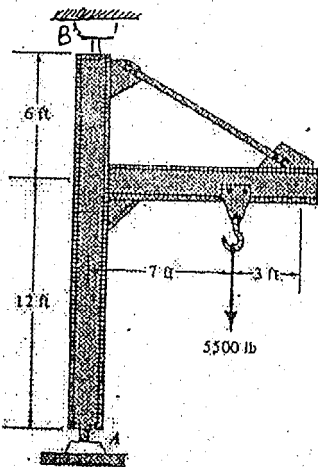
Prob 4-1



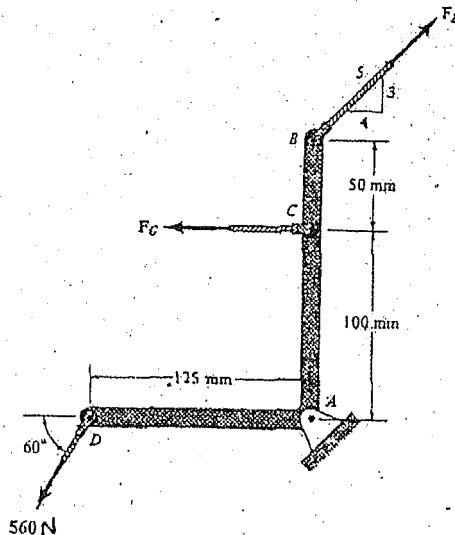
Prob 4-2

4-2 Draw the free-body diagram of member ABC supported by a pin at A and a horizontal short link BD. Explain the significance of each force acting on the diagram.

4-3 The crane is used to support a load of 5500 lb. If B exerts only a horizontal force on the crane, whereas A exerts both horizontal and vertical forces, determine these reactions for equilibrium.



Prob 4-3



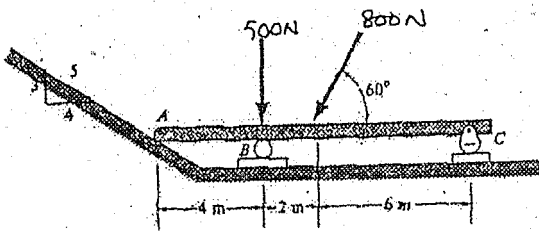
Probs 4-4/4-5

4-4. The link is subjected to the loadings shown. Determine the force  $F_B$  in the cable at B

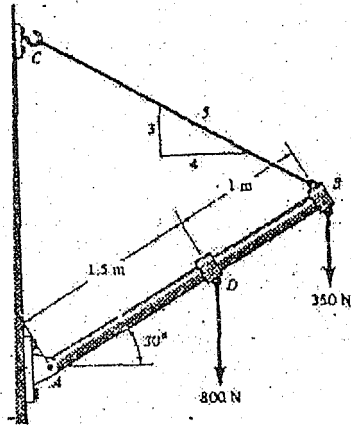
and the magnitude of the resultant force acting on the pin at A. Set  $F_C = 230 \text{ N}$ .

4-5. Determine the forces  $F_B$  and  $F_C$  which can be applied to the link so that the magnitude of the resultant force acting at the pin A is  $850 \text{ N}$ .

4-6. Determine the reactions at the roller B, the rocker C, and where the beam contacts the smooth plane at A. Neglect the thickness of the beam.



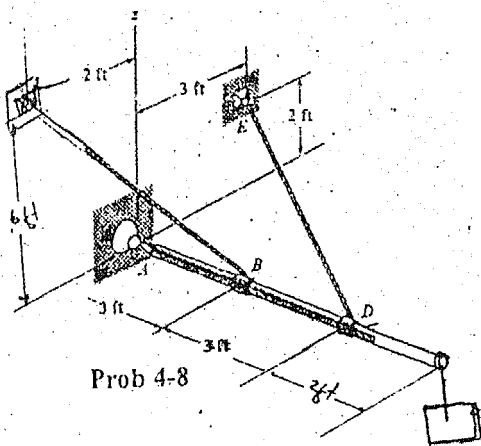
Prob 4-6



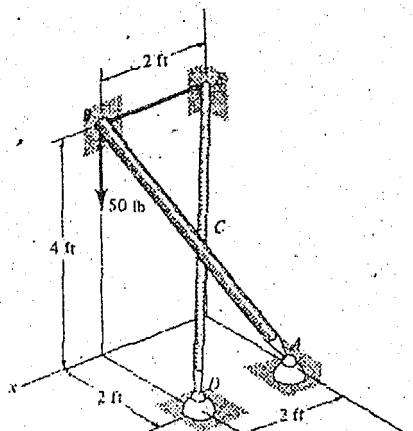
Prob 4-7

4-7. The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB.

4-8. The boom supports a load having a weight of  $W = 850 \text{ lb}$ . Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in cables BC and DE.



Prob 4-8



Prob 4-9

4-9. Rod AB is supported by a ball-and-socket joint at A and a cable at B. Also, the rod rests against the smooth wall at B and against the thin smooth rod at C. Determine the x, y, z components of reaction at these supports if the rod is subjected to a 50 lb vertical force as shown.

**Example 4-7**

Determine the tension in cables  $BC$  and  $BD$  and the reactions at the ball-and-socket joint  $A$  for the mast shown in Fig. 4-9*a*.

**SOLUTION (VECTOR ANALYSIS)**

**Free-Body Diagram.** There are five unknown force magnitudes shown on the free-body diagram, Fig. 4-9*b*.

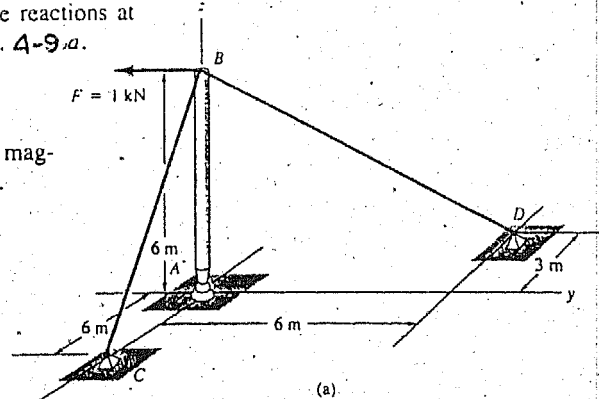
**Equations of Equilibrium.** Expressing each force in Cartesian vector form, we have

$$F = \{-1000\mathbf{j}\} \text{ N}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$T_C = 0.707T_C\mathbf{i} - 0.707T_C\mathbf{k}$$

$$T_D = T_D \left( \frac{\mathbf{r}_{BD}}{r_{BD}} \right) = -0.333T_D\mathbf{i} + 0.667T_D\mathbf{j} - 0.667T_D\mathbf{k}$$



Applying the force equation of equilibrium gives

$$\Sigma F = 0; \quad F + F_A + T_C + T_D = 0$$

$$(A_x + 0.707T_C - 0.333T_D)\mathbf{i} + (-1000 + A_y + 0.667T_D)\mathbf{j} + (A_z - 0.707T_C - 0.667T_D)\mathbf{k} = 0$$

$$\Sigma F_x = 0; \quad A_x + 0.707T_C - 0.333T_D = 0 \quad (1)$$

$$\Sigma F_y = 0; \quad A_y + 0.667T_D - 1000 = 0 \quad (2)$$

$$\Sigma F_z = 0; \quad A_z - 0.707T_C - 0.667T_D = 0 \quad (3)$$

Summing moments about point  $A$ , we have

$$\Sigma M_A = 0; \quad \mathbf{r}_B \times (F + T_C + T_D) = 0$$

$$6\mathbf{k} \times (-1000\mathbf{j} + 0.707T_C\mathbf{i} - 0.707T_C\mathbf{k} - 0.333T_D\mathbf{i} + 0.667T_D\mathbf{j} - 0.667T_D\mathbf{k}) = 0$$

Evaluating the cross-product and combining terms yields

$$(-4T_D + 6000)\mathbf{i} + (4.24T_C - 2T_D)\mathbf{j} = 0$$

$$\Sigma M_x = 0; \quad -4T_D + 6000 = 0 \quad (4)$$

$$\Sigma M_y = 0; \quad 4.24T_C - 2T_D = 0 \quad (5)$$

The moment equation about the  $z$  axis,  $\Sigma M_z = 0$ , is automatically satisfied. Why? Solving Eqs. (1) to (5) we have

$$T_C = 707 \text{ N} \quad T_D = 1500 \text{ N} \quad \text{Ans.}$$

$$A_x = 0.0 \text{ N} \quad A_y = 0.0 \text{ N} \quad A_z = 1500 \text{ N} \quad \text{Ans.}$$

Since the mast is a two-force member, note that the values of  $A_x = A_y = 0.0$  could have been determined by inspection.

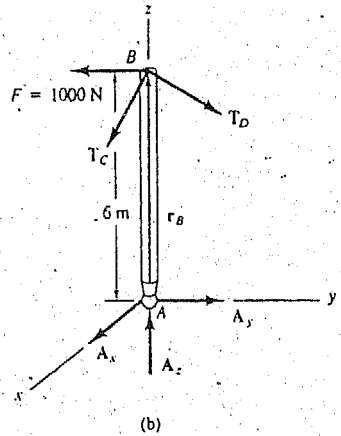


Fig. 4-9

### Example 4-8

Rod  $AB$  shown in Fig. 4-10a is used to support the 200-N force. Determine the reactions at the ball-and-socket joint  $A$  and at the smooth collar  $B$ .

#### SOLUTION (VECTOR ANALYSIS)

**Free-Body Diagram.** Here we have considered the rod and attached collar together as shown in Fig. 4-10b.

**Equations of Equilibrium.** Representing each force on the free-body diagram in Cartesian vector form, we have

$$\mathbf{F}_A = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

$$\mathbf{F}_B = B_x \mathbf{i} + B_y \mathbf{j}$$

$$\mathbf{F} = \{-200\mathbf{k}\} \text{ N}$$

Applying the force equation of equilibrium,

$$\Sigma \mathbf{F} = 0; \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F} = 0$$

$$(A_x + B_x) \mathbf{i} + (A_y + B_y) \mathbf{j} + (A_z - 200) \mathbf{k} = 0$$

$$\Sigma F_x = 0; \quad A_x + B_x = 0 \quad (1)$$

$$\Sigma F_y = 0; \quad A_y + B_y = 0 \quad (2)$$

$$\Sigma F_z = 0; \quad A_z - 200 = 0 \quad (3)$$

Summing moments about point  $A$  yields

$$\Sigma M_A = 0; \quad (\mathbf{r}_C \times \mathbf{F}) + (\mathbf{r}_B \times \mathbf{F}_B) = 0$$

Since  $\mathbf{r}_C = \frac{1}{2} \mathbf{r}_B$ , then

$$(1\mathbf{i} + 1\mathbf{j} - 0.5\mathbf{k}) \times (-200\mathbf{k}) + (2\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}) \times (B_x \mathbf{i} + B_y \mathbf{j}) = 0$$

Expanding and rearranging terms gives

$$(B_y - 200)\mathbf{i} + (-B_x + 200)\mathbf{j} + (2B_y - 2B_x)\mathbf{k} = 0$$

$$\Sigma M_x = 0; \quad B_y - 200 = 0 \quad (4)$$

$$\Sigma M_y = 0; \quad -B_x + 200 = 0 \quad (5)$$

$$\Sigma M_z = 0; \quad 2B_y - 2B_x = 0 \quad (6)$$

Solving Eqs. (1) to (6), we get

$$A_x = A_y = -200 \text{ N} \quad \text{Ans.}$$

$$A_z = B_x = B_y = 200 \text{ N} \quad \text{Ans.}$$

The negative sign indicates that  $A_x$  and  $A_y$  have a sense which is opposite to that shown on the free-body diagram, Fig. 4-10b.

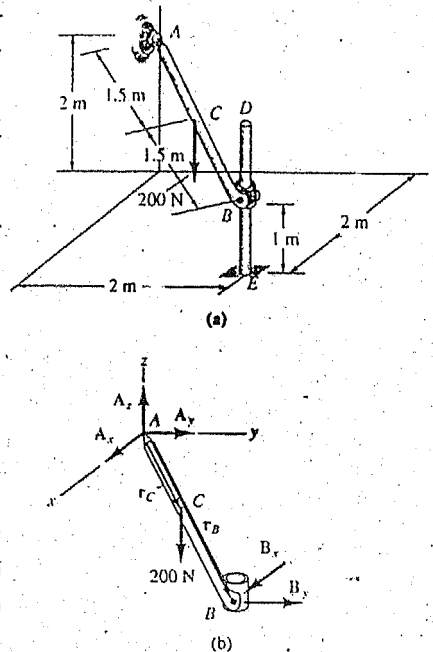
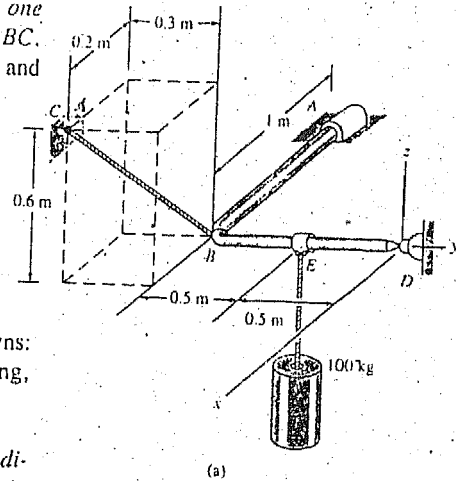


Fig. 4-10

**Example 4-9**

The bent rod in Fig. 4-11a is supported at A by a journal bearing, at D by a ball-and-socket joint, and at B by means of cable BC. Using *one equilibrium equation*, obtain a direct solution for the tension in cable BC. The bearing at A is capable of exerting force components only in the z and y directions, since it is properly aligned on the shaft.



**SOLUTION (VECTOR ANALYSIS)**

**Free-Body Diagram.** As shown in Fig. 4-11b, there are six unknowns: the three force components at the ball-and-socket joint, two at the bearing, and the tension force in the cable.

**Equations of Equilibrium.** The cable tension  $T_B$  may be obtained directly by summing moments about an axis passing through points D and A. Why? The direction of the axis is defined by the unit vector  $u$ , where

$$u = \frac{r_{DA}}{r_{DA}} = -\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$$

$$= -0.707i - 0.707j$$

Hence, the sum of the moments about this axis is zero provided

$$\Sigma M_{DA} = u \cdot \Sigma (r \times F) = 0$$

Here  $r$  represents a position vector drawn from *any point* on the axis DA to any point on the line of action of force  $F$  (see Eq. 3-9). With reference to Fig. 4-11b, we can therefore write

$$u \cdot (r_B \times T_B + r_E \times W) = 0$$

$$(-0.707i - 0.707j) \cdot [(-1j) \times \left(\frac{0.2}{0.7}T_B i - \frac{0.3}{0.7}T_B j + \frac{0.6}{0.7}T_B k\right)$$

$$+ (-0.5j) \times (-981k)] = 0$$

$$(-0.707i - 0.707j) \cdot [(-0.857T_B + 490.5)i + 0.286T_B k] = 0$$

$$-0.707(-0.857T_B + 490.5) + 0 + 0 = 0$$

$$T_B = \frac{490.5}{0.857} = 572 \text{ N}$$

*Ans.*

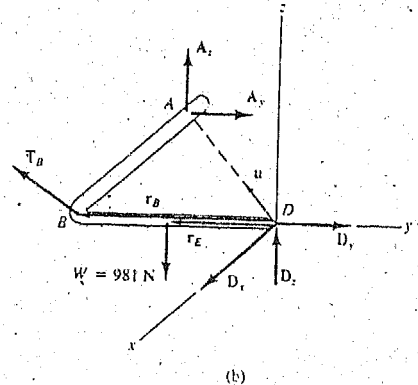
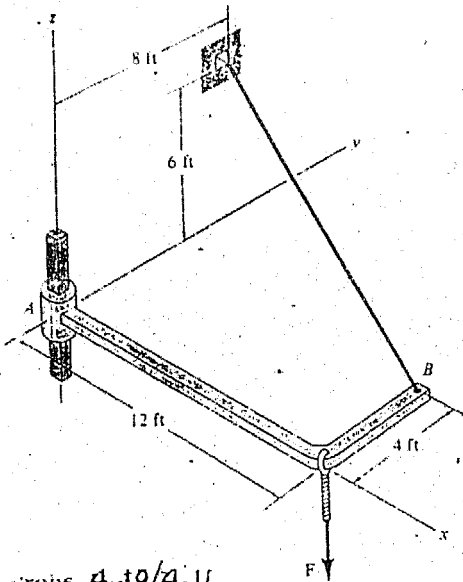


Fig. 4-11

The advantage of using Cartesian vectors for this solution should be noted. It would be especially tedious to determine the perpendicular distance from the DA axis to the line of action of  $T_B$  using scalar methods.

**Note:** In Example 4-7, a direct solution for  $A_z$  is possible by summing moments about an axis passing through the supports at C and D, Fig. 4-11c. If this is done only the moment of  $F$  and  $A_z$  must be considered. Try and apply the above technique and determine the result  $A_z = 1500 \text{ N}$ .

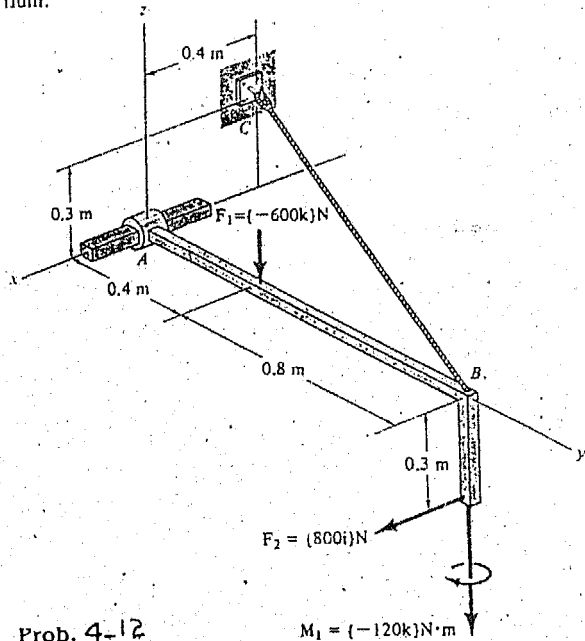
~~4-10~~ Member  $AB$  is supported by a cable  $BC$  and at  $A$  by a smooth fixed square rod which fits loosely through the square hole of the collar. If  $F = \{20i - 40j - 75k\}$  lb, determine the  $x, y, z$  components of reaction at  $A$  and the tension in the cable.



Probs. 4-10/4-11

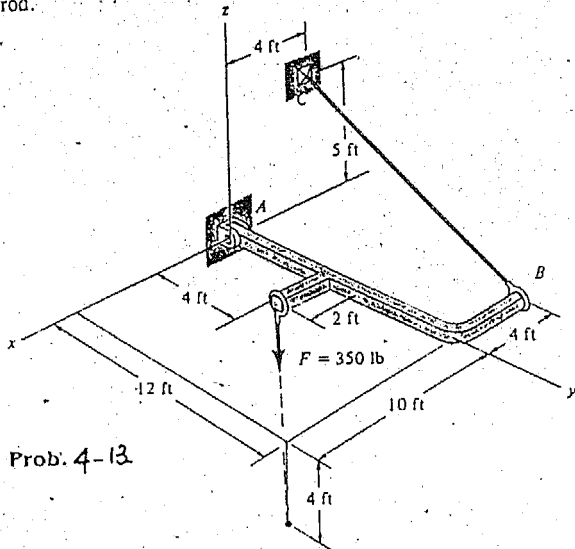
~~4-11~~ Member  $AB$  is supported by a cable  $BC$  and at  $A$  by a smooth fixed square rod which fits loosely through the square hole of the collar. Determine the tension in the cable  $BC$  if the force  $F = -\{45k\}$  lb.

~~4-12~~ The member is supported by cable  $BC$  and at  $A$  by a smooth fixed square rod which fits loosely through the square hole of the collar. Determine the  $x, y, z$  components of reaction at  $A$  and the tension in the cable needed to hold the member in equilibrium.



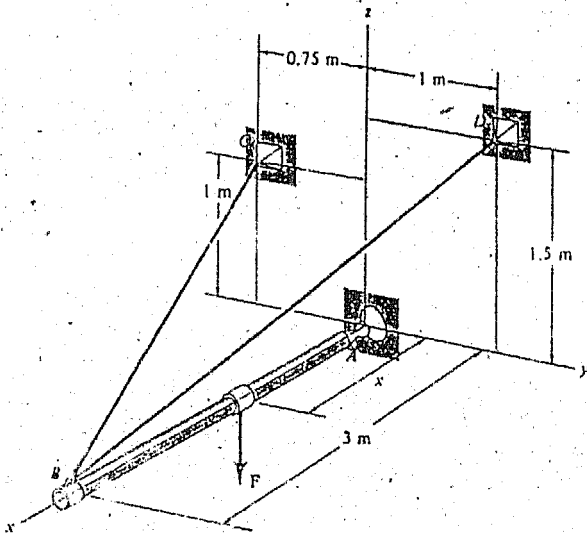
Prob. 4-12

~~4-13~~ Determine the  $x, y, z$  components of reaction at the pin  $A$  and the tension in the cable  $BC$  necessary for equilibrium of the rod.



Prob. 4-13

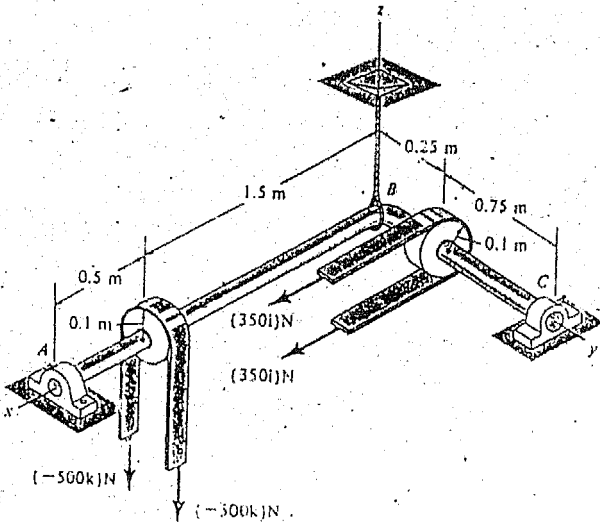
4-14. The boom supports the force of  $F = \{-8k\}$  kN. Determine the  $x, y, z$  components of reaction at the ball-and-socket joint  $A$  and the tension in the two cables  $BC$  and  $BD$ . Set  $x = 2$  m.



Probs. 4-14/4-15

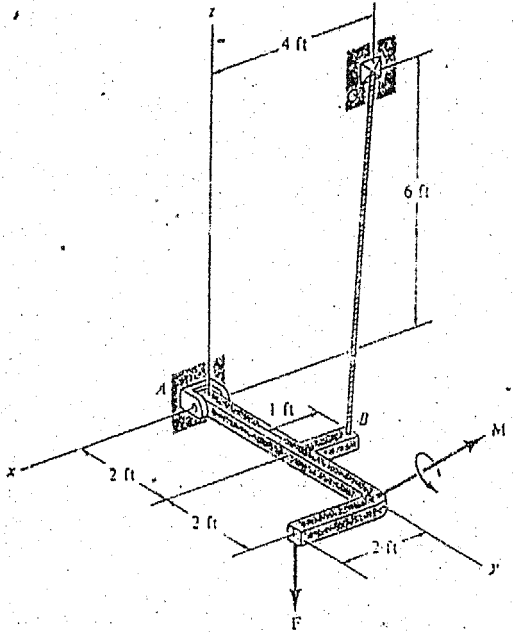
4-15. The boom supports the force of  $F = \{-8k\}$  kN. If cable  $BC$  or  $BD$  can support a maximum tension of 4 kN before it breaks, determine the greatest distance  $x$  the force can be positioned along the boom. Also, determine the  $x, y, z$  components of reaction at the ball-and-socket joint  $A$ .

4-16. The shaft is supported by a thrust bearing at  $A$ , a cable at  $B$ , and a journal bearing at  $C$ . Determine the  $x, y, z$  components of reaction at these supports needed to hold the shaft in equilibrium. The bearings are properly aligned on the shaft.



Prob. 4-16

4-17. The member is supported by cable  $BC$  and at  $A$  by a pin. If  $F = \{50i + 50j - 125k\}$  lb and  $M = \{-400i + 300j + 500k\}$  lb · ft, determine the  $x, y, z$  components of reaction at  $A$  and the tension in the cable.



Probs. 4-17/4-18

4-18. The member is supported by cable  $BC$  and at  $A$  by a pin. If the tension in the cable is to be  $T_{BC} = 250$  lb and  $M = \{-400i + 300j + 500k\}$  lb · ft, determine the magnitude of  $F$  if its coordinate direction angles are  $\alpha = 60^\circ$ ,  $\beta = 60^\circ$ ,  $\gamma = 135^\circ$ . Also, compute the  $x, y, z$  components of reaction at  $A$ .