CHAPTER(2)

Equilibrium of a Particle

In this chapter the methods of resolving a force into components and expressing a force as a Cartesian vector will be used to solve problems involving the equilibrium of a particle. Recall that the dimensions or size of a particle are assumed to be neglected, and therefore a particle can be subjected *only to* a system of *concurrent forces*. To simplify the discussion, the equilibrium of a particle subjected to a coplanar force system will be considered first. Then, in the last part of the chapter, equilibrium problems involving three-dimensional force systems will be considered.

2.1 Condition for the Equilibrium of a Particle

A particle is in *equilibrium* provided it is at rest if originally at rest, or has a constant velocity if originally in motion. Most often, however, the term "equilibrium" or more specifically "static equilibrium" is used to describe an object at rest. To maintain a stale of equilibrium, it is *necessary* to satisfy Newton's first law of motion, which states that if the *resultant force* acting on a particle is *zero*, then the particle is in equilibrium. This condition may be stated mathematically as

$$\sum F = 0 \qquad (2\text{-}1)$$

where $\sum F$ is the vector *sum of all the forces* acting on the particle.

Not only is Eq. 2-1 a necessary condition for equilibrium, but it is also a *sufficient* condition. This follows from Newton's second law of motion, which can be written as $\sum F = ma$. Since the force system satisfies Eq. 2-1, then $ma = 0$, and therefore the particle's acceleration $a = 0$, and consequently the particle indeed moves with constant velocity or remains at rest.

2.2 The Free-Body Diagram

To correctly apply the equation of equilibrium, it is necessary to account for *all* the known and unknown forces ($\sum F$) which act *on* the particle. The best way to do this is to draw the particle's free-body diagram. This diagram is a sketch of the particle which represents it as being isolated or "free" from its surroundings. On this sketch one then shows *all* the forces which the surroundings exert *on* the particle. Provided this diagram is correctly drawn, it will then be easy to apply Eq. 2-1.

Before presenting a formal procedure as to how to draw a free-body diagram, we will first discuss two types of supports often encountered in particle equilibrium problems.

Cables and Pulleys

All cables are assumed to have negligible weight and they cannot be stretched. A cable can support *only* a tension or "pulling" force, and this force always acts in the direction of the cable. Later it will be shown that the tension force developed in a *continuous cable* which passes over a frictionless pulley must have a *constant* magnitude to keep the cable in equilibrium, . Hence, for any angle θ, shown in Fig. 2-1, the cable is subjected to a constant tension T throughout its length.

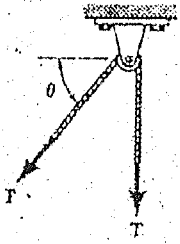


Fig 2-1

Springs

If a *linear elastic spring* is used for a support, the length of the spring will change in direct proportion to the force acting on it. A characteristic that defines the "elasticity" of a spring is the *spring constant* or *stiffness* k. Specifically, the magnitude of force developed by a linear elastic spring which has a stiffness k, and is deformed (compressed or elongated) a distance (measured from its unloaded position, is

$$F = ks \qquad (2\text{-}2)$$

Note that s is determined from the difference in the spring's deformed length l and its undeformed length l_o, i.e., $s = l - l_o$

PROCEDURE FOR DRAWING A FREE-BODY DIAGRAM

To construct a free-body diagram, the following three steps are necessary.

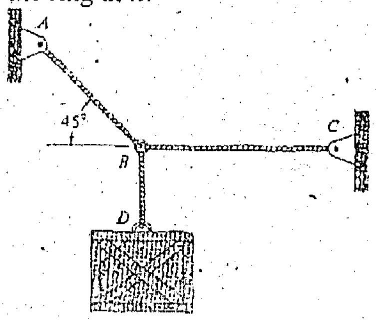
Step 1. Imagine the particle to be *isolated* or cut "free" from its surroundings. Draw or sketch its outlined shape.

Step 2. Indicate on this sketch *all* the forces that act *on the particle*. These forces can be *active forces*, which tend to set the particle in motion, such as those caused by attached cables, weight, or magnetic and electrostatic interaction. Also, *reactive forces* will occur, such as those caused by the constraints or supports that tend to prevent motion. To account for all these forces, it may help to trace around the particle's boundary, carefully noting each force acting on it.

Step 3. The forces that are *known* should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown. In particular, if a force has a known line of action but unknown magnitude, the "arrowhead," which defines the sense of the force, can be *assumed*. The correct sense will become apparent after solving for the unknown magnitude. By definition, the *magnitude* of a force is *always positive* so that, if the solution yields a "negative" scalar, the *minus sign* indicates that the arrowhead or sense of the force is opposite to that which was originally assumed.

Example 2—1

The crate in Fig. 2-2a has a weight of 20 Ib. Draw a free-body diagram of the cord *BD* and the ring at *B*.



SOLUTION

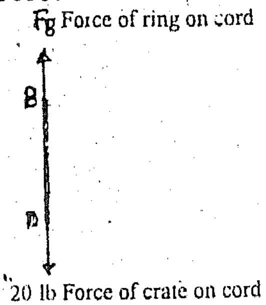
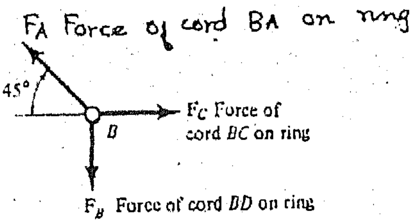


Fig 2-2 (b)



(c)

Example 2—2

The sphere in Fig. 2-3a has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the knot at C.

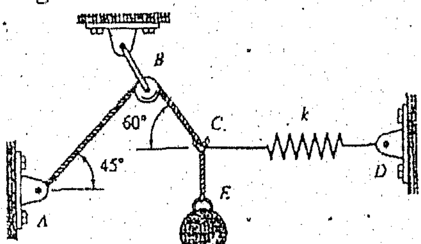
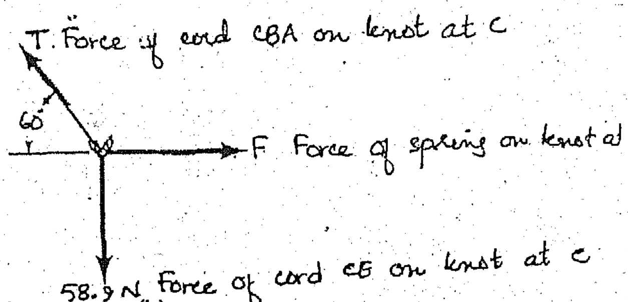


Fig 2-3(a)



(b)

SOLUTION

By inspection three forces act on C; they are caused by cords *CBA* and *CE*, and the spring *CD*. Note that the weight of the sphere, $W = (6\text{ kg})\ (9.81\text{ m/s}^2) = 58.9\text{ N}$, is "transmitted" to point C by the supporting cord *CE*. That is, C is subjected to the force of the cable *CE;* it is not subjected *directly to* the weight of the sphere. Thus, the free-body diagram of the knot at C is shown in Fig.2-3b.

2.3 Coplanar Force Systems

Many particle equilibrium problems involve a coplanar force system. If the forces lie in the *x*-plane, they can each be resolved into their respective i and j components and Eq. 2-1 can be written as

$$\sum F = 0$$

$$\sum F_x i + \sum F_y j \quad 0$$

For this vector equation to be satisfied, both the *x* and y components must equal zero, otherwise $\sum F \neq 0$. Hence, we require

$$\sum F_x = 0$$

$$\sum F_y = 0 \qquad (2\text{-}3)$$

These scalar equilibrium equations state that the algebraic sum of the x and y components of all the forces acting on the particle be equal to zero. As a result, Eqs 2-3 can be solved for at most two unknowns, generally represented as angles and magnitudes of forces shown on the particle's free-body diagram.

Since each of the two equilibrium equations requires the resolution of vector components along a specified axis (x or y), we will use scalar notation to represent the components when applying these equations. By doing this, the sense of direction for each component is accounted for by an *algebraic sign* which corresponds to the arrowhead direction of the component as indicated graphically on the free-body diagram. In particular, if a force component has an unknown magnitude, then the arrowhead sense of the force on the free-body diagram can be *assumed*. Since the magnitude of a force is *always positive*, then if the *solution* yields a *negative scalar*, it indicates that the sense of the force shown on the free-body diagram is opposite to that which was assumed.

These scalar equilibrium equations state that the algebraic sum of the x and y components of all the forces acting on the particle be equal to zero. As a result, Eqs. 2-3 can be solved for at most two unknowns, generally represented as angles and magnitudes of forces shown on the particle's free-body diagram.

PROCEDURE FOR ANALYSIS

The following procedure provides a method for solving coplanar force problems involving particle equilibrium:

Free-Body Diagram. Draw a free-body diagram of the particle This requires that all the known and unknown force magnitudes and angles be labeled on the diagram. The sense of a force having an unknown magnitude can be assumed.

Equations of Equilibrium. Establish the *x*, *y* axes in *any* suitable direction and apply the two equations of equilibrium $\sum F_x = 0$ and $\sum F_y = 0$. For application, components are positive it they are directed along the positive axes, and negative if they are directed along the negative axes. If more than two unknowns exist and the problem involves a spring, apply $F = ks$ (Eq. 2-2) to relate the spring force to the deformation s of the spring

Example2-3

Determine the tension in cables *AB* and *AD* for equilibrium of the 250-kg engine shown in Fig 2-4a.

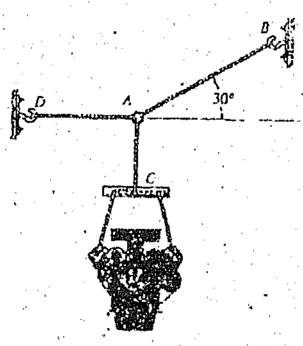
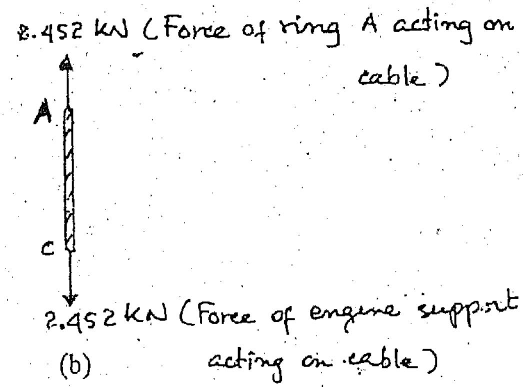


Fig 2-4(a)



(b)

SOLUTION

Free-Body Diagram. To solve this problem we will investigate the equilibrium of the ring at A, because this "particle" is subjected to the forces of both cables *AB* and *AD*.

Equations of Equilibrium The two unknown magnitudes T_B and T_D can be obtained from the two scalar equations of equilibrium, $\sum F_x = 0$ and $\sum F_y = 0$. To apply these equations, the x, y axes are established on the free-body diagram and T_B is resolved into its dashed *x* and *y*

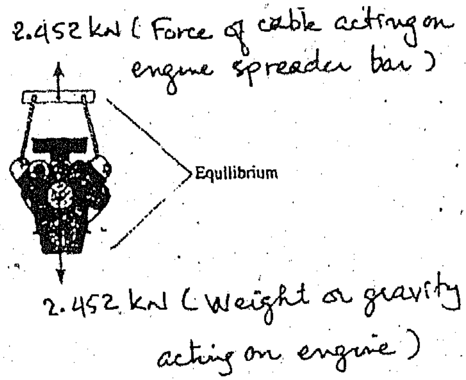
components.
Thus,

$$\xrightarrow{+} \sum F_x = 0; \quad T_B \cos 30^\circ - T_D = 0 \qquad (1)$$

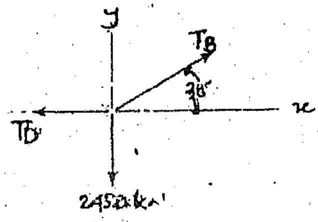
$$+\uparrow \sum F_y = 0; \quad T_B \sin 30^\circ - 2.452kN = 0 \qquad (2)$$

Solving Eq.(2) for T_B

$T_B = 4.91\text{kN}$, $T_D = 4.52$ kN



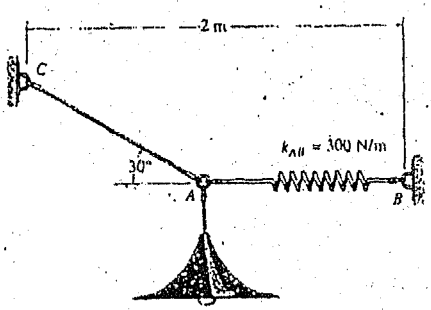
(c)



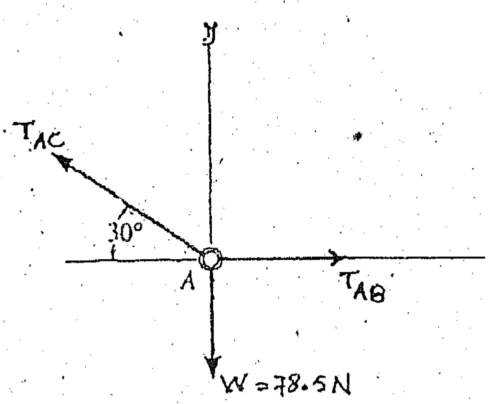
(d)

Example 2-4

Determine the required length of cord AC in Fig. 2-5a so that the 8-kg lamp is suspended in the position shown. The *undeformed* length of the spring AB is $l'_{AB} = 0.4\text{m}$ and the spring has a stiffness of k_{AB} = 300N/m.



(a)



(b)

Fig 2-5

SOLUTION

If the force in spring AB is known, the stretch of the spring can be found ($F = ks$). Using the problem geometry, it is then possible to calculate the required length of AC.
Free-Body Diagram. The lamp has a weight W = 8(9.81) = 78.5 N. The free-body diagram of the ring at A is shown in Fig- 2-5b.

Equations of Equilibrium Using the A. y axes,

$\xrightarrow{+} \sum F_x = 0;$ $T_{AB} - T_{AC} \cos 30° = 0$

$+\uparrow \sum F_y = 0;$ $T_{AC} \sin 30° - 78.5N = 0$

Solving we obtain,

$$T_{AC} = 157.0N$$

$$T_{AB} = 136.0N$$

The stretch of spring AB is therefore

$T_{AB} = k_{AB} s_{AB};$ 136.0 N = 300 N/m (s_{AB})

$$s_{AB} = 0.453 \text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$

$$l_{AB} = 0.4m + 0.453m = 0.853m$$

The horizontal distance from C to B. required

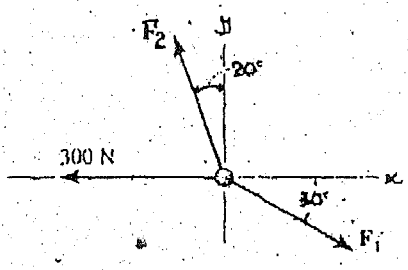
2m = $l_{AC} \cos 30°$ + 0.853 m

l_{AC} = 1.32m

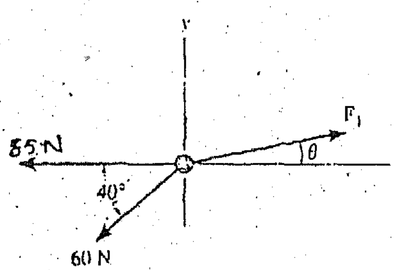
Problems

2-1. Determine the magnitudes of F_1 and F_2 so that the particle is in equilibrium.

2-2. Determine the magnitude and direction θ of F_1 so that the particles is in equilibrium



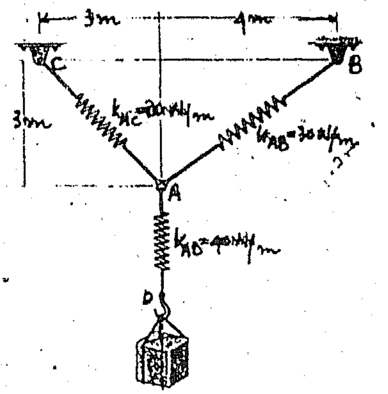
Prob 2-1



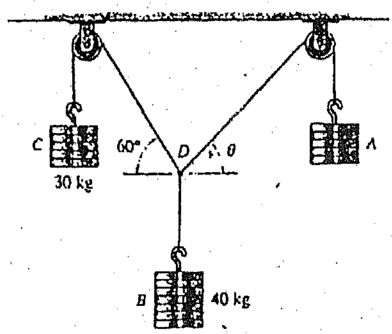
Prob 2-2

2-3. The unstretched length of spring AB is 2 m. If the block held in the equilibrium position shown, determine the mass of the block at D.

2-4. Determine the mass that must be supported at A and the angle θ of the connecting cord in order to hold the system in equilibrium.



Prob2-3

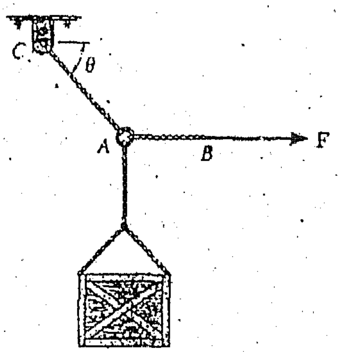


Prob 2-4

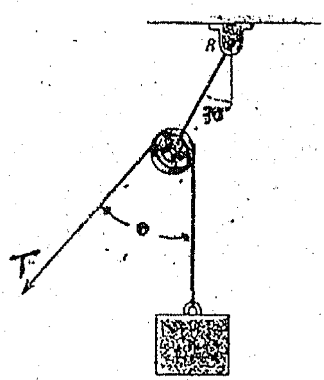
2-5. The 500-N crate is hoisted using the ropes AB and AC. Each rope can withstand a maximum tension of 2500 N before it breaks. If AB always remains horizontal, determine the smallest angle θ to which the crate can be hoisted

2-6. The block has a weigh of 20 N and is being hoisted at uniform velocity. Determine the angle θ for equilibrium and the required force in each cord.

2-7. Determine the maximum weight W of the block that can be suspended in the position shown if each cord can support a maximum tension of 80 N. Also, what is the angle θ for equilibrium



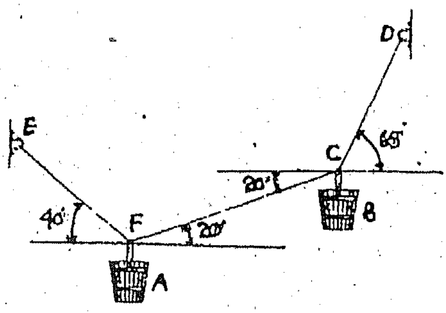
Prob2-5



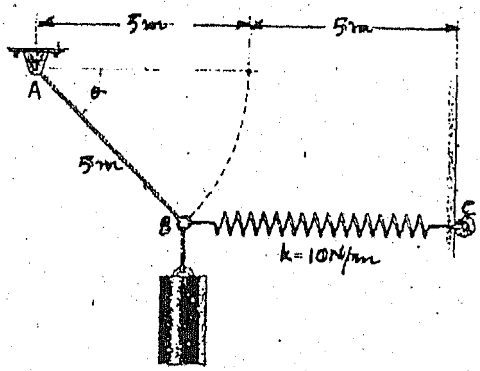
Prob 2-6/2-7

2-8. If the cords suspend the two buckets in the equilibrium position shown, determine the weight of bucket B. Bucket A has a weight of 60 N,

2-9. The cord AB has a length of 5 m and is attached to the end B of the spring having a stiffness k = 10 N/m. The other end of the spring is attached to a roller C so that the spring remains horizontal as it stretches. If a 10-N weight is suspended from B, determine the necessary unstretched length of the spring so that θ = 40° for equilibrium



Prob2-8



Prob 2-9

2.4 Three-Dimensional Force Systems

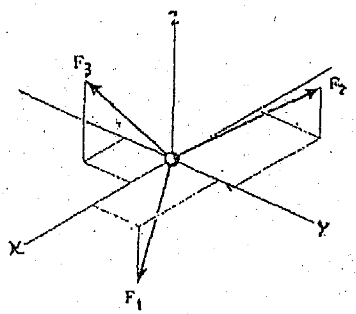


Fig 2-6

It was shown in Sec. 2.1 that panicle equilibrium requires

$$\sum F \quad 0 \qquad (2\text{-}4)$$

If the forces acting on the particle are resolved into their respective **i, j, k** components Fig, 2-6, we can then write

$$\sum F_x i + \sum F_y j + \sum F_z k = 0$$

To ensure that Eq. 2-4 is satisfied, we must therefore require that the following three scalar component equations be satisfied:

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0 \qquad (2\text{-}5)$$

These equations represent the algebraic sums of the *x, y,* z force components, acting on the particle. Using them we can solve for at most three unknowns generally represented as angles or magnitudes of forces shown on the particle's free-body diagram.

PROCEDURE FOR ANALYSIS

The following, procedure provides a method for solving three dimensional force equilibrium problems.

Free-Body Diagram, Draw a free-body diagram of the particle and label all the known and unknown forces on this diagram.

Equations of Equilibrium. Establish the *x, y, z* coordinate axes with origin located at the particle and apply the equations of equilibrium. Use the three scalar Eqs; 2-5 in cases where it is easy to resolve each forces acting on the particle into its x, y, z components.

If this appears difficult, first express each force acting on the particle in Cartesian vector form, and then substitute these vectors into Eq. 2-4. By setting the respective i, j, k components equal to zero, the three scalar Eqs. 2-5 can be generated. If more than three unknowns exist and the problem involves a spring, consider using $F = ks$ to relate the spring force to the deformation s of the spring.

Example 2-5

A 90-N load is suspended from the hook shown in Fig.2-7a. The load is supported by two cables and a spring having a stiffness $k = 500$ N/m Determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x-y plane and cable AC lies in the x-z plane

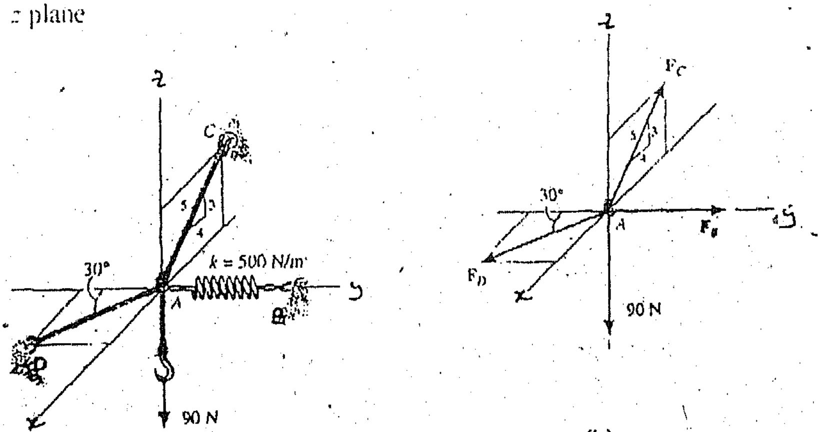


Fig 2-7 (a) (b)

SOLUTION

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig.2-7b.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components. and therefore the three scalar equations of equilibrium can be directly applied. Considering components directed along the positive axes as "positive," we have

$$\sum F_x = 0; \quad F_D \sin 30° - \frac{4}{5} F_C = 0 \quad (1)$$

$$\sum F_y = 0 \quad -F_D \cos 30° + F_B = 0 \quad (2)$$

$$\sum F_z = 0; \quad \frac{3}{5} F_C - 90 \text{ N} = 0 \quad (3)$$

Solving Eq 3 for F_C, then Eq. 1 for F_D and finally Eq. 2 for F_B we get

$F_C = 150$N

$F_D = 240$N

$F_B = 208$N

The stretch of the spring is therefore

$$F_B = ks_{AB}$$

$$208 \text{ N} = 500 \text{ N/m} \left(s_{AB}\right)$$

$$s_{AB} = 0.416 \text{ m}$$

Example 2-6

Determine the force developed in each cable used to support the 40-N crate shown in Fig. 2-8a.

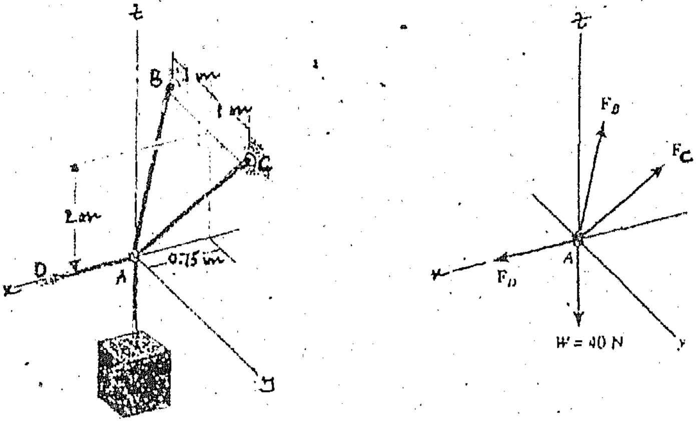


Fig 2-8 (a) (b)

SOLUTION

Free-Body Diagram. As shown in Fig.2-8b, the free-body diagram of point A is considered in order to "expose" the three unknown forces in the cables, and by applying the condition for equilibrium we can obtain their magnitudes.

Equations of Equilibrium. First we will express each force in Cartesian vector form. Since the coordinates of points B and C are B (- 0.75 m, -1 m, 2 m,) and C (-0.75 m, 1 m, 2 m,), we have

$$F_B = F_B\left[\frac{-0.75i - 1j + 2k}{\sqrt{(-0.75)^2 + (-1)^2 + (2)^2}}\right]$$

$$= -0.318F_B i - 0.424F_B j + 0.848F_B k$$

$$F_C = F_C\left[\frac{-0.75i + 1j + 2k}{\sqrt{(-0.75)^2 + (1)^2 + (2)^2}}\right]$$

$$= -0.318F_C i + 0.424F_C j + 0.848F_C k$$

$$F_D = F_D i$$

$$W = \{-40k\}N$$

Equilibrium requires

$$\sum F = 0; \quad F_B + F_C + F_D + W = 0$$

$$0.318F_B i - 0.424F_B j + 0.848F_B k - 0.318F_C i + 0.424F_C j + 0.848F_C k + F_D i - 40k$$

Equating the respective i, j, k components to zero yields

$\sum F_x$ 0; $\quad -0.318F_B - 0.318F_C + F_D = 0 \quad (1)$

$\sum F_y$ 0; $\quad -0.424F_B + 0.424F_C = 0 \quad (2)$

$\sum F_z$ 0; $\quad 0.848F_B + 0.848F_C - 40 \quad 0 \quad (3)$

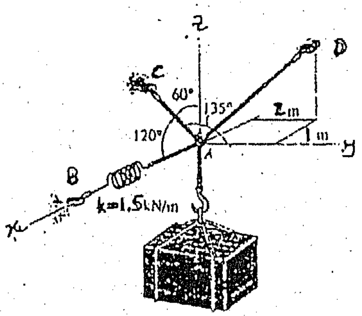
Equation 2 states that $F_B = F_C$ Thus, solving Eq. 3 for F_B and F_C and substituting the result into Eq 1 to obtain F_D, we have

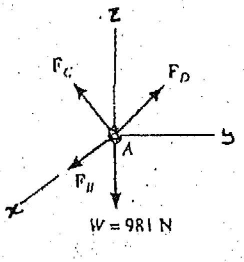
$F_B = F_C = 23.6$ N

$F_D = 15.0$N

Example 2-7

The 100-kg box shown in Fig. 2-9(a) is supported by three cords- one of which is connected to a spring. Determine the tension in each cord and the stretch of the spring





(b)

Fig 2-9(a)

SOLUTION

Free-Body Diagram. The force in each of the cords can be determined by investigating the equilibrium of point *A*. The tree-body diagram is shown in Fig. 2-9b. The weight of the cylinder is W = 100(9.81) =981 N.

Equations of Equilibrium. Each vector on the free-body diagram is first expressed in Cartesian vector form-for F_C, and noting point D(-1 m, 2m, 2 m) for F_D, we have

$$F_B = F_B i$$

$$F_C = F_C \cos 120° i + F_C \cos 135° j + F_C \cos 60° k$$

$$= -0.5F_C i - 0.707F_C j + 0.5F_C k$$

$$F_D = F_D\left[\frac{-1i + 2j + 2k}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}\right]$$

$= -0.333F_D i + 0.667F_D j + 0.667F_D k$

$W = \{-981k\}$ N

Equilibrium requires

$\sum F = 0; \qquad F_B + F_C + F_D + W = 0$

$F_B i - 0.5F_C i - 0.707F_C j + 0.5F_C k - 0.333F_D i + 0.667F_D j + 0.667F_D k - 981k$
0

Equating the respective i, j, k components to zero,

$\sum F_x = 0; \quad F_B - 0.5F_C - 0.333F_D = 0 \qquad (1)$

$\sum F_y = 0; \quad -0.707F_C + 0.667F_D = 0 \qquad (2)$

$\sum F_z = 0; \quad 0.5F_C + 0.667F_D - 981 = 0 \qquad (3)$

Solving Eq 2 for F_D in terms of F_C and substituting into Eq 3 yields F_C. F_D is determined from Eq 2. Finally, substituting the results into Eq. 1 yields F_B. Hence.

$F_C = 813$ N

$F_D = 862$ N

$F_B = 693.7$ N

The stretch of the spring is therefore

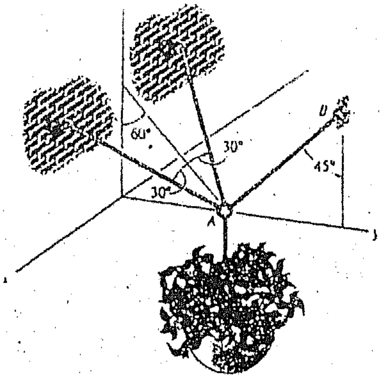
F=ks; 639.7=1500s

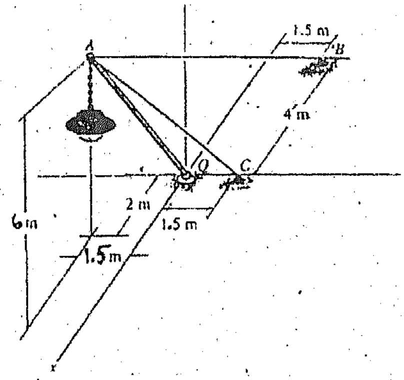
s = 0.462 m

Problems

2-10. The 25-kg flowerpot is supported at A by the three cords. Determine the force acting in each cord for equilibrium.

2-11. If each cord can sustain a maximum tension of 50 N before it fails, determine the greatest weight of the flowerpot the cords can support





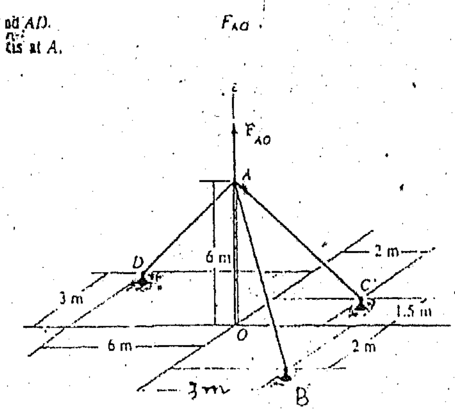
Probs 2-10/2-11

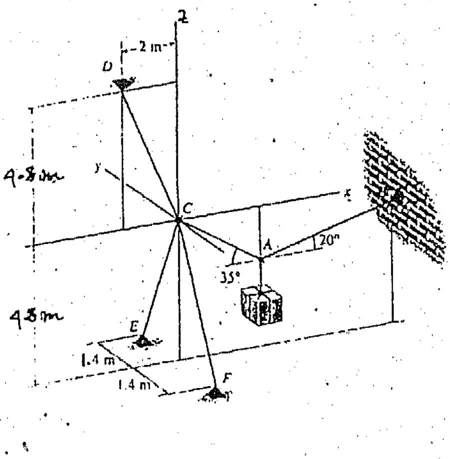
Probs 2-12/2-13

2-12. The lamp has a mass of 15 kg and is supported by a pole AO and cables AB and AC. If the force in the pole acts along its axis, determine the forces in AO, AB, and AC for equilibrium.

2-13. Cables AB and AC can sustain a maximum tension of 500N, and the pole can support a maximum compression of 300N. Determine the maximum weight of the lamp that can be supported in the position shown. The force in the pole acts along the axis of the pole

2-14. The mast OA is supported by three cables. If cable AB is subjected to a tension of 500 N, determine the tension in cables AC and AD and the vertical force F_{AO} which the mast exerts along its axis at A.





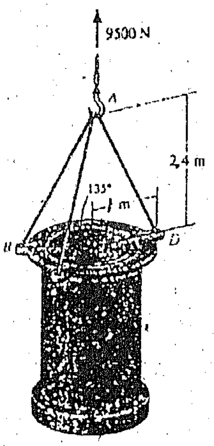
Prob. 2-14.

Prob 2-15

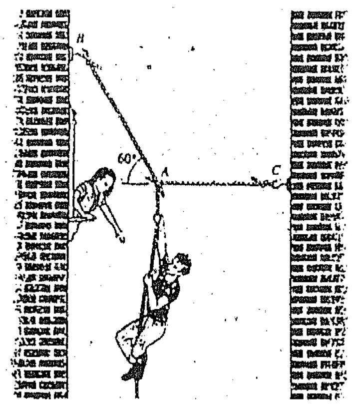
2-15. The 500-N crate is suspended from the cable system shown. Determine the force in each segment of the cable, i.e., AC, CD, CE, and CF. *Hint:* First analyze the equilibrium of point A, then using the result for AC, analyze the equilibrium of point C.

2-16. The 9500-N crucible is supported by three cables. Determine the force in each cable for equilibrium of the hook at A.

2-17. Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point *A*. Any of the three segments of the rope can sustain a maximum force of 2 kN before it breaks. Determine if Romeo, who has a mass of 65 kg, can climb the rope, and if so, can he along with his Juliet, who has a mass of 60 kg, climb down with constant velocity?



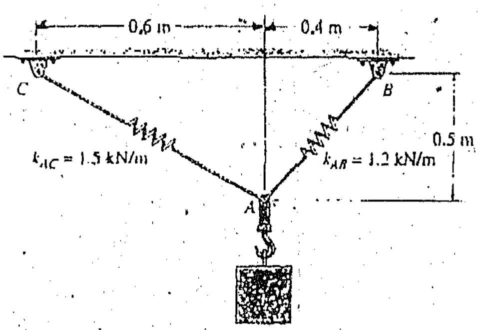
Prob 2-16



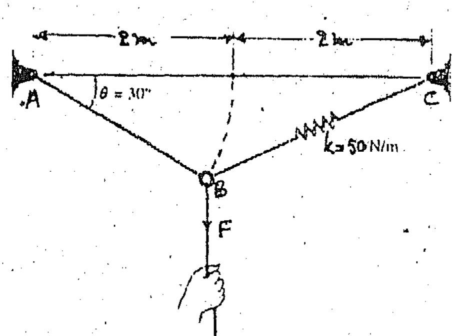
Prob 2-17

2-18. The 30-kg block is supported by two springs having the stiffness shown. Determine the unstretched length of each spring after the block is removed.

2-19. The 2 m long cord AB is attached to a spring BC having an unstretched length of 2 m. If the cord sags downward and amount $\theta = 30°$ as shown, determine the vertical force F applied. The *spring* has a stiffness of $k = 50$N/m.



Prob 2-18



Prob 2-19