CHAPTER(9)
Kinetics of a Particle:

Force and Acceleration

In Chapter 8 we developed the methods needed to formulate the acceleration of a particle in terms of its velocity and position. In this chapter we will use these concepts when applying Newton's second law of motion, F = *ma*, to study the effects caused by an unbalanced force acting on a particle. Depending upon the geometry of the path, the analysis of problems will be performed using rectangular, normal and tangential, or cylindrical coordinates. In the last part of the chapter, Newton's second law of motion will be used to study problems in space mechanics.

9-1 The Equation of Motion

When more than one force acts on a particle, the resultant force is determined by a vector summation of all the forces; i.e., $F_R = \sum F$. For this more general case, the equation of motion may be written as

$$\sum F = ma. \qquad (9\text{-}1)$$

In particular, note that if $F_R = \sum F = 0$, then the acceleration is also zero, so that the particle will either remain at *rest* or move along a straight-line path with *constant velocity*. Such are the conditions of *static equilibrium*, Newton's first law of motion.

9-2 Equation of motion (rectangular coordinate)

When a particle is moving relative to an inertial *x, y, z* frame of reference, the forces acting on the particle, as well as its acceleration, may be expressed in terms of their **i, j**, k components, Fig. 9-1. Applying the equation of motion, we have

$$\sum F = ma$$

$$\sum F_x i + \sum F_y j + \sum F_z k = m\left(a_x i + a_y j + a_z k\right)$$

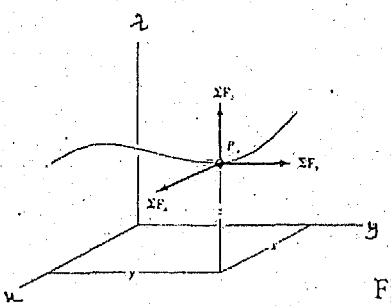


Fig 9-1

For this equation to be satisfied, the respective i, j, and k components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations;

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y \qquad (9\text{-}2)$$
$$\sum F_z = ma_z$$

In particular, if the particle is constrained to move in the *x-y* plane, then only the first two of these equations are used to specify the motion.

PROCEDURE FOR ANALYSIS

The equations of motion are used to solve problems which require a relationship between the forces and the accelerated motion they cause. Whenever they are applied, the unknown force and acceleration components should be identified and an equivalent number of equations should be written. If further equations are necessary for the solution, kinematics may be considered. Recall that these equations only relate the geometric properties of the motion, and therefore they become useful if the particle's position or velocity is to be related to its acceleration. Whatever the situation, the following procedure provides a general method for solving problems in kinetics.

Free-Body Diagram. Select the inertial coordinate system. Most often, rectangular *x, y, z* coordinates are chosen to analyze problems for which the particle has *rectilinear motion.* If this occurs, one of the axes should extend in the direction of motion. Once the coordinates are established, draw the particle's free-body diagram. Drawing the free-body diagram is *very important* since it provides a graphical representation *of all the forces* $\left(\sum F\right)$ which act on the particle and thereby makes it possible to resolve these forces into their A, *y*, z components. The direction and sense of the particle's acceleration a should also be established. If the sense of its components is unknown, for mathematical convenience assume that they are in the same *direction* as the *positive* inertial coordinate axes. The acceleration may be sketched on the *x, y, z* coordinate system, *but not on* the free-body diagram, or it may be represented as the ma vector on the kinetic diagram.* Once the free-body diagram has been constructed, identify the unknowns under consideration.

Equations of Motion. If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form. If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can he used for the solution.

Friction If the particle contacts a rough surface, it may be necessary to use *the frictional equation,* which relates the coefficient of kinetic friction μ_k to the magnitudes of the frictional and normal forces F_f and N acting at the surfaces of contact, i.e., $F_f = \mu_k N$..

Spring If the particle is connected to an *elastic spring* having negligible mass, the spring force F_s can be related to the deformation of the spring by the equation $F_s = ks$. Here k is the spring's stiffness measured as a force per unit length, and s is the stretch or compression defined as the difference between the deformed length l and the undeformed length l_0. i.e., $s = l - l_0$.

Kinematics. As stated above, if a complete solution cannot be obtained strictly from the equation of motion, the equations of kinematics may be considered. For example, if the velocity or position of the particle is to be found, it will be necessary to apply the proper kinematic equations once the particle's acceleration is determined from $\sum F = $ ma.

If the *acceleration is a function of time,* use a = dv/dt and *v* = dsldt which, when integrated yield the particle's velocity and position.

If the *acceleration is a function of displacement,* integrate *a ds* = *v dv* to obtain the velocit

as a function of position.
If the *acceleration is constant*, use

$$v = v_0 + a_c t \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$ to determine the

position or velocity of the particle.

In all cases, make sure the positive coordinate directions used/or writing the kinematic equations are the same as those used for writing the equations of motion; otherwise, simultaneous solution of the equations will result in errors. Also, if the solution for an unknown vector component yields a negative scalar, it indicates that the component acts in the direction opposite to that which was assumed

Example 9-1

The 50-kg crate shown in Fig. 9-2a rests on a horizontal plane for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 5 s starting from rest.

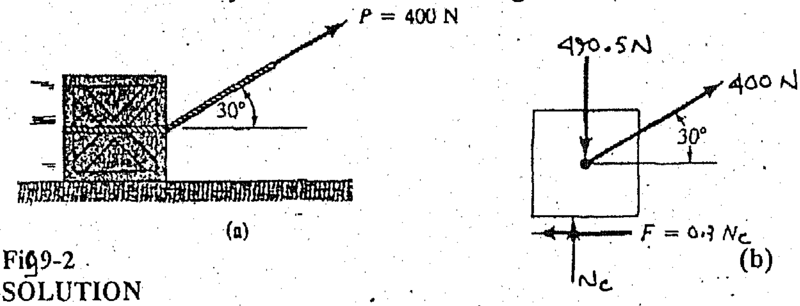


Fig9-2

SOLUTION

This problem requires application of the equations of motion, since the crate's acceleration can be related to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg$ = 50 kg (9.81 m/s²) = 490.5 N. As shown in Fig. 9-2b, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration a is assumed to act horizontal, in the positive x direction. There are two unknowns, namely Nc and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

$$\xrightarrow{+} \sum F_x = ma_x; \quad 400\cos 30° - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow \sum F_y = ma_y; \quad N_C - 490.5 + 400\sin 30° = 0 \qquad (2)$$

Solving Eq. (2) for Nc, substituting the result into Eq. (1), and solving for a yields

Nc = 290.5 N

a = 5.19 m/s²

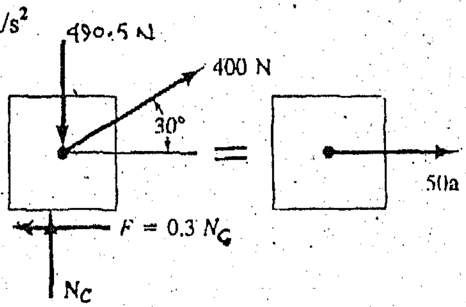


Fig. 9.2 (c)

Kinematics. Since the acceleration is *constant*, and the initial velocity is zero, the velocity of the crate in 5 s is

$(\xrightarrow{+})$ $v = v_0 + a_c t$

$= 0 + 5.19\,(5)$

$= 26.0$ m/s $\rightarrow$

Example 9-2

A smooth 2-kg collar C, shown in Fig. 9-3a, is attached to a spring having a stiffness $k = 3$ N/m and an unstretched length of 0.75 m. If the ; collar is released from rest at *A*, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1$ m.

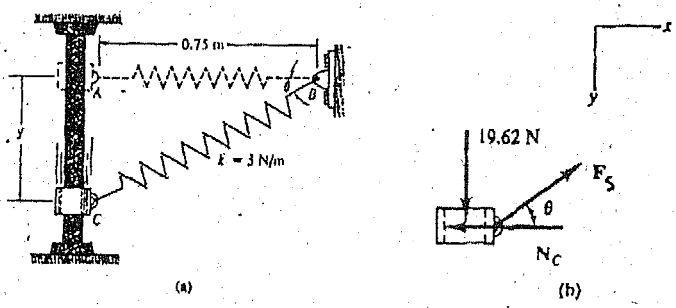


Fig 9-3

SOLUTION

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position *y* is shown in Fig. 9-3b. Note that the weight is $W = 2(9.81) = 19.62$ N. Furthermore, the collar is *assumed* to be accelerating so that 'a' acts downward in the *positive y* direction. There are four unknowns, namely N_c, F_s, a, and θ.

Equations of Motion. Using the data inFig.9-3b,

$$\xrightarrow{+} \sum F_x = ma_x, \qquad -N_c + F_s \cos\theta = 0 \qquad (1)$$

$$+\uparrow \sum F_y = ma_y; \qquad 19.62 - F_s \sin\theta = 2a \qquad (2)$$

From Eq. (2) it is seen that the acceleration is not constant; rather, it depends upon the magnitude and direction of the spring force. Solution is possible once F_s and θ are known.

The magnitude of the spring force is a function of the stretch s of the spring; i.e., $F_s = ks$. Here the unstretched length is $AB = 0.75$ m, Fig. 9-3a; therefore,

$s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75$. Since $k = 3$ N/m, then

$$F_s = ks = 3\left(\sqrt{y^2 + (0.75)^2} - 0.75\right) \qquad (3)$$

From Fig. 9-3a, the angle θ is related to *y* by trigonometry.

$$\sin\theta = \frac{y}{\sqrt{y^2 + (0.75)^2}} \qquad (4)$$

Substituting $y = 1$ m into Eqs. (3) and (4) yields $F_s = 1.50$ N and $\theta = 53.1°$. Substituting these results into Eqs. (1) and (2), we obtain

$$N_C = 0.900N$$

$a = 9.21$ m/s $\downarrow$

Example 9-3

The 100-kgblockA shown in Fig. 9-4 a is released from rest. If the mass of the pulleys and the cord is neglected, determine the speed of the 20-kg block B in 2 s.

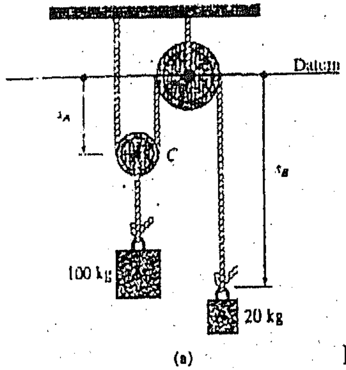
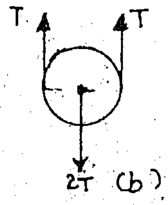


Fig 9-4

SOLUTION

Motion of blocks A and B will be analyzed separately.

Free-Body Diagram. Since the mass of the pulleys is neglected, the equilibrium condition for pulley C is shown in Fig. 9-4b. The free-body diagrams for blocks A and B are shown in Fig. 9- 4c' and d, respectively. Here we *assume* both blocks accelerate downward, in the direction of $+s_A$ and $+s_B$. What are the three unknowns?



Equations of Motion Block A (Fig. 9-4c):

$+\downarrow \sum F_y = ma_y;$ $981 - 2T = 100\, a_A$ (1)

Block B (Fig. 13-10d):

$+\downarrow \sum F_y = ma_y,$ $196.2 - T = 20 a_B$ (2)

Kinematics. The necessary third equation is obtained by studying the kinematics of the pulley arrangement in order to relate a_A to a_B. the coordinates s_A and s_B measure the positions of A and B from the fixed datum, Fig. 9-4a. It is seen that

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

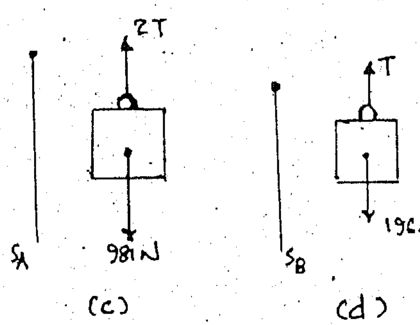
$$2a_A = -a_B \qquad (3)$$

Hence when block A accelerates *downward*, block B accelerates *upward* at twice the amount.

T= 327.0N

a_A=3.27 m/s^2

a_B= -6.54 m/s^2



Since a_B is constant, the velocity of block B in 2 s is thus

$(+\downarrow)$ $$v = v_0 + a_B t$$

$$= 0 + (-6.54)(2)$$

$$= -13.1 \text{m/s}$$

The negative sign indicates that block B is moving upward.

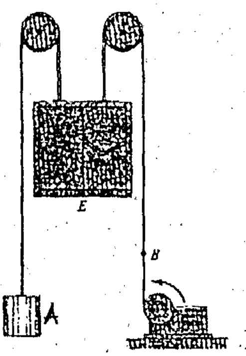
Problems

9-1. The elevator E has a mass of 500 kg and the counter weighl at A has a mass of 150 kg. If the motor supplies a constant force of 5 kN on the cable at B, determine the speed of the elevator in t = 3s starting from rest. Neglect the mass of the pulleys and cable.

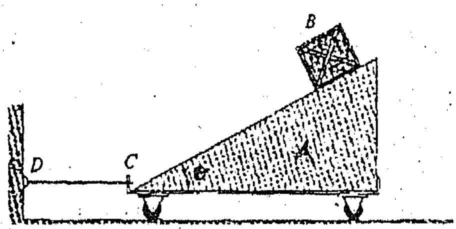
9-2. The elevator E has a mass of 500 kg and the counterweight at A has a mass of 150 kg. If the elevator attains a speed of 10 m/s after it rises 40 m, determine the force developed in the cable at B. Neglect the mass of the pulleys and cable.

9-3. Block B has a mass m and is released from rest when it is on top of cart A, which has a mass of 3 m. Determine the tension in cord CD needed to hold the cart from moving while B is sliding down A. Neglect friction.

9-4. Block B has a mass m and is released from rest when it is on top of cart A, which has amass of 3 m. Determine the tension in cord CD needed to hold the cart from moving while B is sliding down A. The coefficient of kinetic friction between A and B is μ.

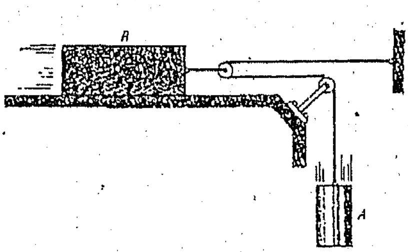


Probs. 9-1/9-2

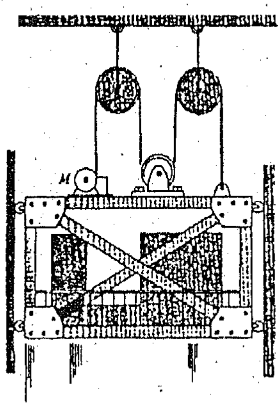


Probs. 9-3/9-

9-5. At a given instant the 5-lb block A is moving downward with a speed of 4 ft/s. Determine its speed 3 s later. Block B has a weight of 6 Ib, and the coefficient of kinetic friction between it and the horizontal plane is $\mu_k = 0.3$. Neglect the mass of the pulleys and cord.



Prob 9-5

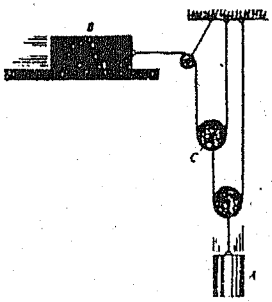


Probs 9-6/9-7

9-6. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by using the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 1.50$ kN in its attached cable, determine the velocity of the elevator when it has moved upward 3 m starting from rest. Neglect the mass of the pulleys and cables.

9-7. A freight elevator, including its load, has a mass of 500 kg. It is prevented from rotating by using the track and wheels mounted along its sides. Starting from rest, in t = 2 s, the motor M draws in the cable with a speed of 6 m/s, *measured relative to the elevator.* Determine the constant acceleration of the elevator and the tension in the cable. Neglect the mass of the pulleys and cables.

9-8. Determine the acceleration of the 5-kg cylinder A. Neglect the mass of the pulleys and cords. The block at B has a mass of 10 kg. Assume the surface at B is smooth.

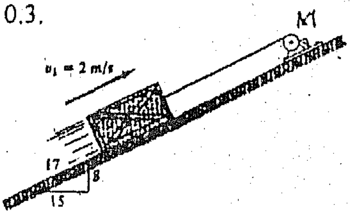


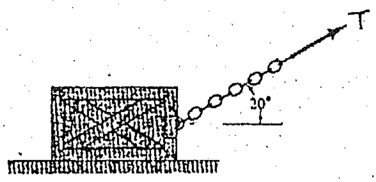
Probs. 9-8/9-9

9-9. Determine the acceleration of the 5-kg cylinder A. Neglect the mass of the pulleys and cords. The block at B has a mass of 10 kg. The coefficient of kinetic friction between block B and the surface is $\mu_k = 0.1$.

9-10. The 100-kg crate is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = 800t^2$ N, where t is in seconds. If the crate has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when t = 2 s. The coefficient of kinetic friction between the crate and the incline is $\mu_k = 0.3$.

9-11. The 100-kg crate is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = 800t^2$ N, where t is in seconds. If the crate has an initial velocity $v_1 = 2$ m/s at $s = 0$ and t = 0, determine the distance the crate moves up the plane when t = 2 s. The coefficient of kinetic friction between the crate and the incline is $\mu_k = 0.3$.





Probs 9-10/9-11

Probs 9-12/9-13

9-12. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of T is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.

9-13. The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in t = 2 s if the coefficient of static friction is $\mu_s = 0.4$ and the coefficient of kinetic friction is $\mu_k = 0.3$ and the towing

CHAPTER (10)

Kinetics of a Particle:

Work and Energy

In this chapter we will integrate the equation of motion with respect to displacement and thereby obtain the principle of work and energy. This principle is useful for solving problems which involve force, velocity, and displacement. Later in the chapter the concept of power is discussed and a method is presented for solving kinetic problems using the theorem of conservation of energy. Before presenting these topics, however, it is first necessary to define the work done by various types of forces.

10.1 The Work of a Force

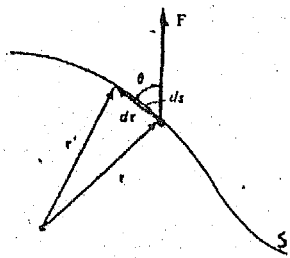


Fig 10.1

In mechanics a force F does *work* only when it undergoes a *displacement in the direction of the force*. For example, consider the force F having a location on the path *s* which is specified by the position vector **r**, Fig. 10-1. If the force moves along the path to a new position **r'**, the displacement is then dt = r'-r. The magnitude of dr is represented by *ds*, the differential segment along the path. If the angle between the tails of dr and F is θ, Fig. 10-1, then the work *dU* which is done by F is a *scalar quantity*, defined by

$$dU = F\, ds \cos\theta$$

By definition of the dot product (see Eq. C-14) the above equation may also be written as

$$dU = \mathbf{F} \cdot d\mathbf{r}$$

Work as expressed by the above equation may be interpreted in one of two ways: either as the product of F and the component of displacement in the direction of the force, i.e., $ds \cos\theta$, or as the product of ds and the component of force in the direction of displacement, i.e., $F \cos\theta$

Note that if $0^\circ \leq \theta \leq 90^\circ$, then the force component and the displacement have the *same sense so* that the work is *positive;* whereas if $90^\circ < \theta \leq 180^\circ$, these vectors have an *opposite sense* and therefore the work is *negative*. Also, $dU = 0$ if the force is *perpendicular* to displacement, since $\cos 90^\circ = 0$, or if the force is applied at *a fixed point*, in which case the displacement is zero.

The basic unit for work in the SI system is called a joule (J). This unit combines the units of force and displacement. Specifically, 1 *joule of* work is done when a force of 1 newton moves 1 meter along its line of action (1 J = 1 N • m). The moment of a force has this same combination of units (N • m); however, the concepts of moment and work are in no way related. A moment is a vector quantity, whereas work is a scalar. In the FPS system work is generally defined by writing the units as ft • lb, which is distinguished from the units for a moment written as lb • ft.

Work of a Variable Force

If a force undergoes a finite displacement along its path from r_1 to r_2 or s_1 to s_2 Fig. 10-2a, the work is determined by integration. If F is expressed as a function of position, $F = F(s)$, we have

$$U_{1-2} = \int_{r_1}^{r_2} F \bullet dr = \int_{s_1}^{s_2} F \cos\theta ds \qquad (10.1)$$

If the working component of the force, $F\cos\theta$, is plotted versus s. Fig. 10-2b, the integral represented in this equation can be interpreted as the *area under the curve* from position s_1 to position s_2

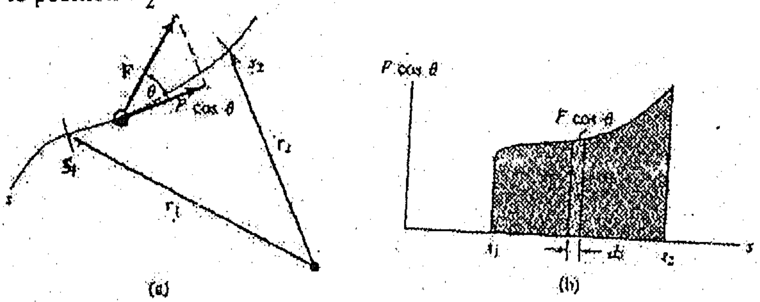


Fig 10-2

Work of a Constant Force Moving Along a Straight Line

If the force F_c has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig. 10-3a, then the component of F_c in the direction of displacement is $F_c \cos\theta$. The work done by F_c when it is displaced from s_1, to s_2 is determined by Eq. 10-1, in which case

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

or,

$$U_{1-2} = F_c \cos\theta (s_2 - s_1) \qquad (10\text{-}2)$$

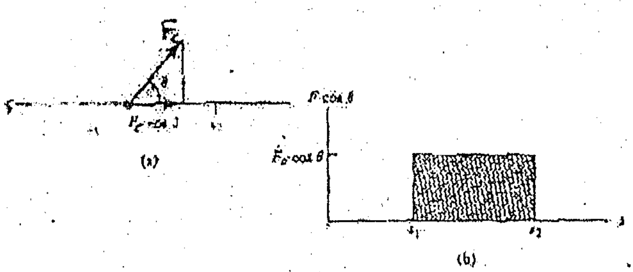


Fig 10-3

Here the work of F_c represents the *area under the rectangle* in Fig. 10-3b.

Work of a Weight

Consider a particle which moves up along the path.(shown in Fig. 10-4 from position s_1 to position s_2. At an intermediate point, the displacement $dr = dx\,\mathrm{i} + dy\,\mathrm{j} + dz\,\mathrm{k}$. Since W = -Wj, applying Eq. 10-1 yields

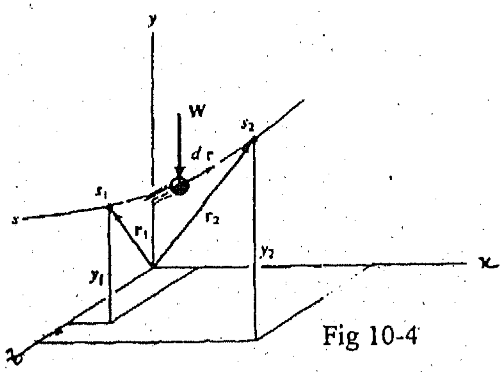


Fig 10-4

$$U_{1\text{-}2} = \int \mathrm{F} \cdot dr = \int_{r_1}^{r_2} (-W\mathrm{j}) \cdot (dx\, i + dy\,\mathrm{j} + dz\,\mathrm{k})$$

$$= \int_{y_1}^{y_2} -Wdy = -W(y_2 - y_1)$$

or,

$$U_{1-2} = -W\Delta y \qquad (10\text{-}3)$$

Thus, the work done is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig. 10-4 the work is *negative,* since W is downward and Δ y is upward. Note, however, that if the particle is displaced *downward* (-Δ y) the work of the weight is *positive.*

Work of a Spring Force

The magnitude of force developed in a linear elastic spring when the spring is displaced a distance i from its unstretched position is $F_s = ks$, where k is the spring stiffness. If the spring is elongated or compressed from a position s, to a further position s_2. Fig. 14-5a, the work done *on the spring by* F_s is *positive,* since in each case the force and displacement are in the *same direction.* We require

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F_s \; ds = \int_{s_1}^{s_2} ks\, ds$$

$$= \frac{1}{2}ks_2^{\,2} - \frac{1}{2}ks_1^{\,2}$$

This equation represents the trapezoidal area under the line $F_s = ks$ versus.s, Fig. 10-'o.

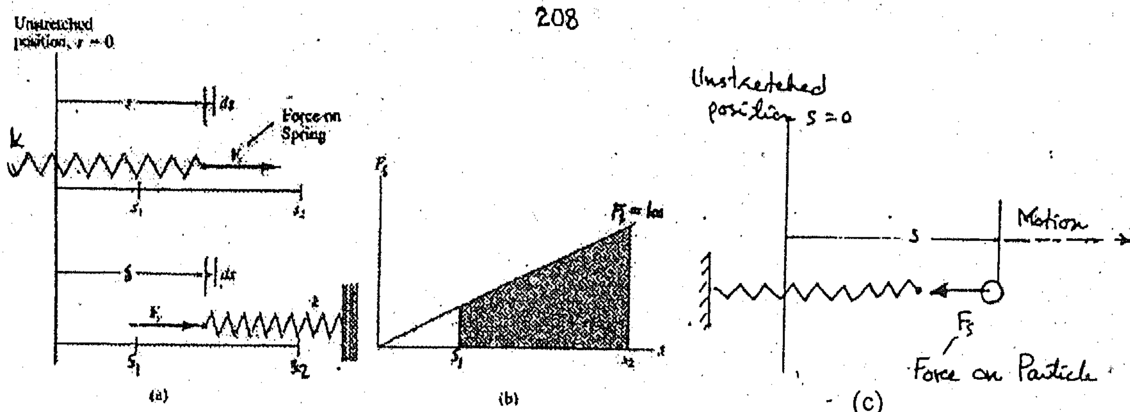


Fig 10-5

If a particle (or body) is attached to a spring, then the force F, exerted on the particle is *opposite* to that exerted on the spring. Consequently, the force will do *negative work* on the particle when the particle is moving so as to further elongate (or compress) the spring. Hence, the above equation becomes

$$U_{1-2} = -\left(\frac{1}{2}ks_2^{\,2} - \frac{1}{2}ks_1^{\,2}\right) \qquad (10\text{-}4)$$

When this equation is used, a mistake in sign can be eliminated if one simply notes the direction of the spring force acting on the particle and compares it with the direction of displacement of the particle—if both are in the *same direction, positive work* results; if they arc *opposite to* one another, the *work is negative*

Example 10-1

The 10-kg block shown in Fig. 10-6a rests on the smooth incline. If the spring is originally unstretched, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.

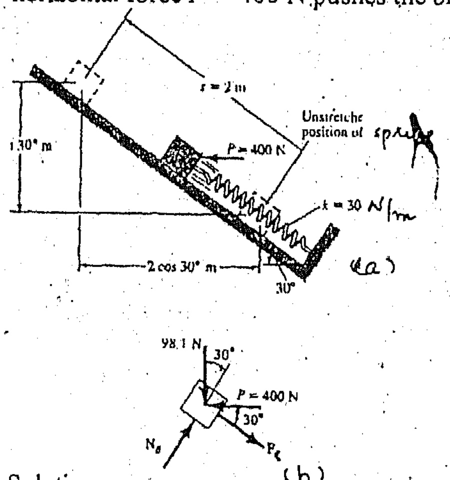


Fig. 10-6

Solution

The free-body diagram of the block is shown in Fig. 10-6b.

Horizontal Force **P.** Since the force is *constant,* the work is computed

The result can be calculated as the force times the component of displacement in the direction of the force, i.e.,

$$U_P = 400(2\cos 30^\circ) = 692.8 \text{ J}$$

or, the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400\cos 30^\circ(2) = 692.8 \text{ J}$$

Spring Force F_s. Since the spring is originally unstretched and in the final position is stretched 2m, the work of F_s is

$$U_s = -\frac{1}{2}(30)\,(2)^2 = -60\ J$$

Why is the work negative?

Weight W. Since the weight acts in the opposite direction to its vertical displacement the work is negative; i.e.,

$$U_W = -98.1(2 \sin 30°) = -98.1\ J$$

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1 \sin 30°)\,2 = -98.1\ J$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

The work of all the forces when the block is displaced 2 m is thus

$U_T = 692.8 - 60 - 98.1 = 535$ J *Ans.*

10.2 Principle of Work and Energy

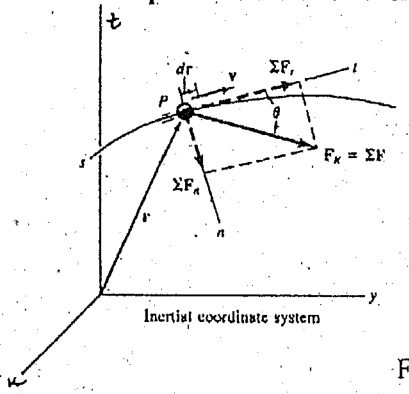


Fig 10-7

Consider a particle *P in* Fig. 10-7, which at the instant considered is located at position r on the path as measured from an inertial coordinate system. If the particle has a mass *m* and is subjected to a system of external forces represented by the resultant $F_R = \sum F$, then the equation of motion for the particle is $\sum F =$ ma. When the particle undergoes a displacement *dr* along the path, the work done by the forces is

$$\sum F \cdot dr = ma \cdot dr$$

If we establish the *n* and t axes at the particle, then $\sum F$ can be resolved into its normal and tangential components. Fig. 10-7. Realizing that the magnitude of *dr* is *ds*, it can be seen that $\sum F \cdot dr = \sum F ds \cos\theta = \sum F_t\ ds$. In other words, the work of2F is computed *only from its tangential components*. The normal components $\sum F_n = \sum F \sin\theta$ do *no work* since they cannot move in the normal direction. Since a • *dt* = $a_t\ ds$, the above equation can also be written as

Using the kinematic equation $a_t\,ds = v\,dv$, and integrating both sides, assuming initially that the particle has a position r = r_1 and a speed $v = v_1$ and later r=r_2, $v = v_2$ yields

$$\sum\int_{r_1}^{r_2} F \bullet dr = \int_{v_1}^{v_2} mvdv$$

or,

$$\sum\int_{r_1}^{r_2} F \bullet dr = \frac{1}{2}mv_2{}^2 - mv_1{}^2 \qquad (10\text{-}5)$$

Using Eq. 10-1, the final result may be written as

$$\sum U_{1-2} = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2 \qquad (10.6)$$

This equation represents the *principle of work and energy* for the particle. The term on the left is the sum of the work done by *all* the forces acting on the particle as the particle moves from point 1 to point 2. The two terms on the right side, which are of the form $T = \frac{1}{2}mv^2$, define the particle's final and initial *kinetic energy,* respectively. These terms are *positive* scalar quantities since they do not depend on the direction of the particle's velocity. Furthermore, Eq. 10-6 is dimensionally homogeneous so that the kinetic energy has the same units as work, e.g., joules (J) or ft • lb.

When Eq. 10-6 is applied, it is often symbolized in the form

$$T_1 + \sum U_{1-2} = T_2 \qquad (10\text{-}7)$$

which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to its final position is equal to the particle's final kinetic energy.

PROCEDURE FOR ANALYSIS

As stated above, the principle of work and energy is used to solve kinetic problems that involve *velocity, force,* and *displacement,* since these terms are involved in the equation. For application it is suggested that the following procedure be used.

Work (Free-Body Diagram). Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that do work on the particle as it moves along its path.

Principle of Work and Energy. Apply the principle of work and energy, $T_1 + \sum U_{1-2} = T_2$ The kinetic energy at the initial and final points is always positive, since it involves the speed squared ($T = \frac{1}{2}mv^2$). The work done by each force shown on the free-body diagram is computed by using the appropriate equations developed in Sec. 10.1. Since *algebraic addition* of the work terms is required, it is important that the proper sign of each term be specified. Specifically, work is positive when the force component is in the same direction as its displacement, otherwise it is negative.

10-3 Principle of Work and Energy for a System of Particles

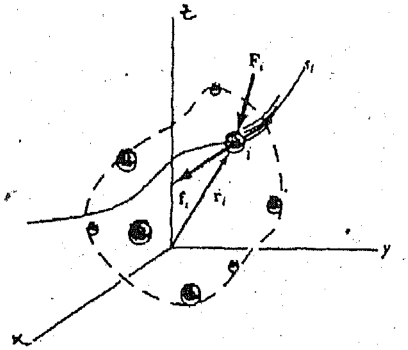


Fig 10-8

The principle of work and energy can be extended to include a system of n particles isolated within an enclosed region of space as shown in Fig. 10-8

We can write this equation symbolically as

$$\sum T_1 + \sum U_{1-2} = \sum T_2 \qquad (10\text{-}8)$$

Hence, the system's initial kinetic energy $\sum T_1$ plus the work done by all the external and internal forces acting on the particles of the system $\sum U_{1-2}$ is equal to the system's final kinetic energy $\sum T_2$. To maintain this balance of energy, strict accountability of the work done by all the forces must be made. In this regard note that although the internal forces on adjacent particles occur in equal but opposite collinear pairs, the total work done by each of these forces will, in general, *not cancel* out since the paths over which corresponding particles travel will be *different*. There are, however, two important exceptions to this rule which often occur in practice. If the particles are contained within the boundary of a *translating rigid body*, the internal forces all undergo displacement by an equal amount, and therefore the internal work will be zero. Also, particles connected by inextensible cables make up a system which has internal forces which arc displaced by an equal amount. In this case adjacent particles exert equal but opposite internal forces which have components that undergo the same displacement, and therefore the work of these forces cancels. On the other hand, note that if the body is assumed to be nonrigid, the particles of the body are displaced along *different paths*, and some of the energy due to force interactions would be given off and lost as heat or stored in the body if permanent deformations occur.

Work of Friction Caused by Sliding

These problems all involve cases where a body is sliding over the surface of another body in the presence of friction. Consider, for example, a block which is translating a distance s over a rough surface as shown in Fig. 10-9. If the applied force P just balances the *resultant* frictional force $\mu_k N$, then due to equilibrium a constant velocity v is maintained and one would expect Eq. 10-8 to be applied as follows

$$\frac{1}{2}mv^2 + Ps - \mu_k Ns = \frac{1}{2}mv^2$$

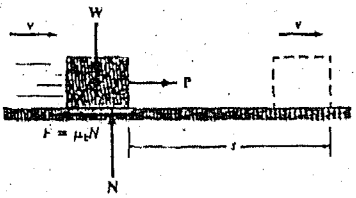


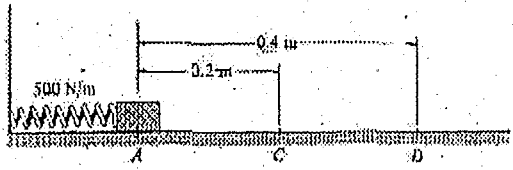
Fig. 14–9 Fig 10-9

Indeed this equation is satisfied if $P = \mu_k N$, however, as one realizes from experience, the sliding motion will *generate heat,* a form of energy which seems not to be accounted fo in the work-energy equation. In order to explain this paradox and thereby more accurately represent the nature of friction, we should actually consider a model of the surfaces of contact which are *deformable* (nonrigid).* Recall that the rough portions at the bottom of the block act as "teeth," and when the block slides these teeth *deform slightly* and either break off or vibrate due to interlocking effects and pulling away from "teeth" at the contacting surface As a result, frictional forces that act at these points are diminished, however, they are replaced by other frictional forces as other points of contact are made. At any instant, the *resultant* of the frictional forces remains essentially constant, i.e., $\mu_k N$; however, due to the localized deformations, the actual displacement s' of $\mu_k N$ is *not* the same displacement s as the applied force P. Instead, s' will be *less* than s *($s' < s$), and* therefore the *external work* done by the resultant frictional force will be $\mu_k Ns'$. The remaining amount of work, $\mu_k N(s - s')$. manifests itself as an increase in *internal energy,* which in fact causes the block's temperature to rise.

In summary then, Eq. 10-8 can be applied to problems involving sliding friction; however, it should he fully realized that the work of the resultant frictional force is not represented by $\mu_k Ns$; instead, this term represents both the external work of friction (μ_k Ns') and internal work[$\mu_k N$ (s - s')] which is converted into various forms of internal energy.!

Example 10-2

A 10-kg block rests on the horizontal surface shown in Fig. 10-10 *a.* The spring, which is not attached to the block, has a stiffness $k = 500$ N/m and S is initially compressed 0.2m from C to A. After the block is released from rest at A, determine its velocity when it passes point D. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



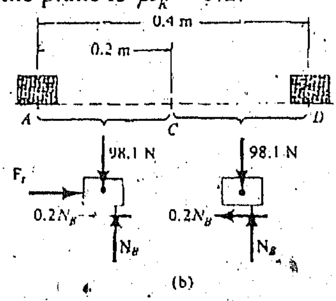


Fig. 10-10 (a)

SOLUTION Why can this problem be solved using the principle of work and energy?

Work (Free-Body Diagrams). Two free-body diagrams for the block are shown in Fig. 10-10b. The block moves under the influence of the spring , force F, along the 0.2-m-long path AC; after which it continues to slide along the plane to point D. With reference to either free-body diagram,

$\sum F_y = 0$; hence, $N_B = 98.1$ N. Only the spring and friction forces do work during the displacement—the spring force does positive work from A to C, whereas the frictional force does negative work.

Principle of Work and Energy

$$T_A + \sum U_{A-D} = T_D$$

$$\frac{1}{2}m(v_A)^2 + \frac{1}{2}ks_{AC}^{\ 2} - 0.2N_B(s_{AD}) = \frac{1}{2}m(v_D)^2$$

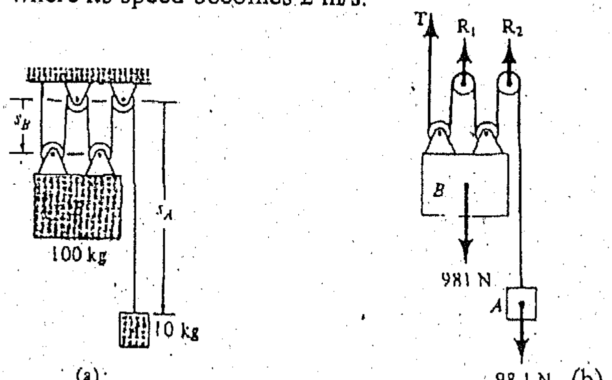
$$\{0\} + \left\{\frac{1}{2}(500)(0.2)^2 - 0.2(9.81)(0.4)\right\} = \left\{\frac{1}{2}(10)(v_D)^2\right\}$$

Solving for v_D, we get

$$v_D = 0.565 m/s$$

Example 10-3

The blocks A and B shown in Fig. 10-11a have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels from the point where it is released from rest to the point where its speed becomes 2 m/s.



(b) Fig 10-11

Solution

Work (Free-Body Diagram). As shown on the free-body diagram of the system. Fig. 10-11b, the cable force T and reactions R_1 and R_2 do *no work,* since these forces represent the reactions at the supports and consequently do not move while the blocks are being displaced.

Principle of Work and Energy. Realizing the blocks are released from rest, and as stated previously, assuming *both* move *downward,* we have

$$\sum T_1 + \sum U_{1-2} = \sum T_2$$

$$\left\{\frac{1}{2}m_A(v_A)_1^{\ 2} + \frac{1}{2}m(v_B)_1^{\ 2}\right\} + \{W_B \Delta s_B + W_A \Delta s_A\} = \left\{\frac{1}{2}m_A(v_A)_2^{\ 2} + \frac{1}{2}m_B(v_B)_2^{\ 2}\right\}$$

$$\{0+0\} + \{981(\Delta s_B) + 98.1(\Delta s_A)\} = \left\{\frac{1}{2}(10)(v_A)_2^{\ 2} + \frac{1}{2}(100)(2)^2\right\} \qquad (1)$$

Kinematics. Using the methods of kinematics, it may be seen from Fig. 10-11a that at any given instant the total length l of all the vertical segments of cable may be expressed in terms of the position coordinates s_A and s_B as $s_A + 4s_B = l$

Hence, a change in position yields the displacement equation

$$\Delta s_A + 4\Delta s_B = 0$$
$$\Delta s_A = -4\Delta s_B \qquad (2)$$

Taking the time derivative yields

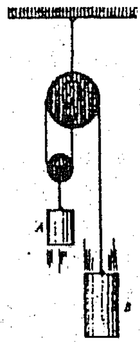
$v_A = -4v_B = -4(2) = -8m/s$

Retaining the negative sign in Eq. (2) and substituting into Eq. (1) yield $\Delta s_B = 0.883$ m ↓

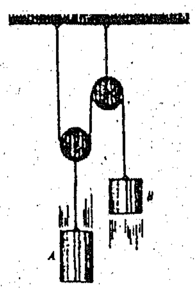
Problems

10-1. Cylinder A has a mass of $m_A = 3$ kg and cylinder B has a mass of $m_B = 8$ kg. Determine the speed of A after it moves upward 2 m starting from rest. Neglect the mass of the cord and pulleys.

10-2. Block A has a weight of $W_A = 60$ Ib and block B has a weight of $W_B = 10$ Ib. Determine the speed of A just after ii moves downward 3 ft starting from rest. Neglect the mass of the cord and pulleys.

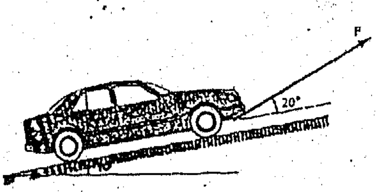


Prob 10-1

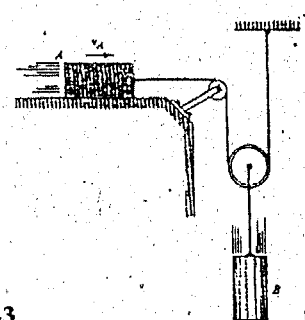


Prob 10 -2

10-3. The car having a mass of 2 Mg is towed along an inclined road. If the car starts from rest and attains a speed of 5 m/s after traveling a distance of 150 m, determine the constant towing force F applied to the car. Neglect friction and the mass of the wheels.



Prob. 10-3

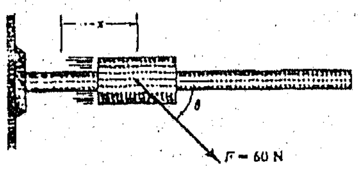


prob 10-4

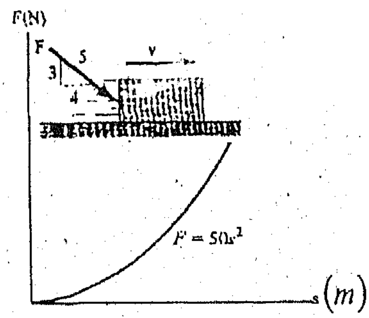
10-4. Block A has a weight of $W_A = 3$ Ib and rests on a surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. Block B has a weight of $W_B = 8$ lb. Determine the distance B must descend so that A has a speed of $v_A = 5$ ft/s starting from rest.

10-5, The collar has a mass of 5 kg and is moving at 8 m/s when $x = 0$. A force of F= -60Nis applied to it as shown. The direction Q of this force varies such that $\theta = 10x$, where x is in meters and θ is clockwise, measured in degrees. Determine the speed of the collar when $x = 3$ m. The coefficient of kinetic friction between the collar and the rod is $\mu_k = 0.3$

10-6. The force F acting in a constant direction on the 20-kg block has a magnitude which varies with the position s of the block. Determine how far the block slides before its velocity becomes 5 m/s. When $s = 0$ the block is moving to the right at 2 m/s. The coefficient of kinetic friction between the block and surface is $\mu_k = 0.3$.



Prob. 10-5



Prob. 10-6