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Q 3.28 A girl on ice skates is at rest on a horizontal sheet of smooth ice. As a result of catching a rubber ball moving horizontally toward her, she moves at  $2 \text{ m s}^{-1}$ . Give a rough estimate of what her speed would have been

- if the rubber ball were thrown twice as fast.
- if the rubber ball had twice the mass.
- if the girl had twice the mass
- if the rubber ball were not caught by the girl, but bounced off and went straight back with no change of speed.

Total initial momentum of the system is  $mv$  of the ball since the girl is at rest.

Final momentum of the system will be taken as momentum of  $MV$  of the girl taking the mass of the ball to be negligible compared to that of the girl.

Conservation of momentum, then, gives,  $MV = mv$ .

(a) When the ball was thrown twice as fast,  $mv$  doubles, double  $MV$  making the girl move twice as fast at  $4 \text{ m s}^{-1}$ .

(b) Doubling the mass of the ball doubles the initial and hence the final momentum giving the girl a speed of  $4 \text{ m s}^{-1}$  again.

(c) If the girl has twice the mass, her speed would be half of the previous, i.e.  $1 \text{ m s}^{-1}$  since  $MV = mv$  the same.

(d) If the rubber ball bounced off, the change in momentum of ball is  $2mv$  or twice that before. For momentum to be conserved, the girl's momentum has also got to change by the same amount, i.e.  $2Mv$  making her move at  $4 \text{ m s}^{-1}$ .

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P3.9 A rifle bullet travelling at  $360 \text{ m s}^{-1}$ , strikes a block of soft wood which it penetrates to a depth of  $10 \text{ cm}$ . The mass of the bullet is  $18 \text{ g}$ . Assume a constant retarding force. (a) How long a time was required for the bullet to stop? (b) What was the decelerating force.

Solution:

$$\text{At } t=0, x_0 = 0, v_0 = 360 \text{ m s}^{-1}$$

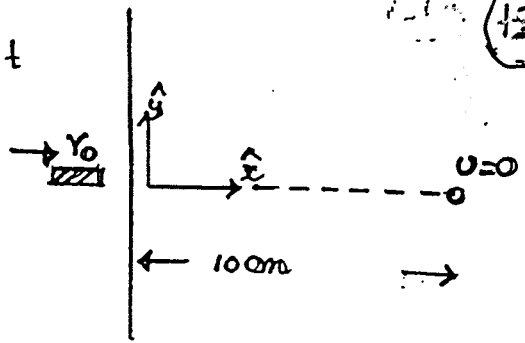
$$\text{at } t = t, x = 10 \text{ cm} = 10^{-1} \text{ m}, v = 0$$

$$m = 1.8 \text{ g} = 1.8 \times 10^{-3} \text{ kg}$$

(a) Since  $x = v_{av} t = \frac{v + v_0}{2} t$

$$10^1 = \frac{360 + 0}{2} t$$

$$t = 0.5 \text{ ms}$$



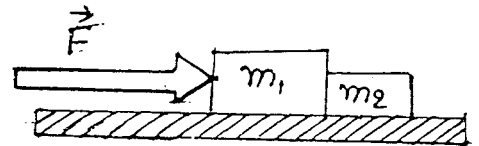
(b)  $F_{net} = ma$

$$= m \frac{\Delta v}{\Delta t} = 1.8 \times 10^{-3} \frac{(0 - 360)}{0.5 \times 10^{-3}} = -1296 \text{ N}$$

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P 3.12. Two blocks are in contact on a frictionless table. A horizontal force is applied to one block, as shown in figure. (a) if  $m_1 = 2 \text{ kg}$ ,  $m_2 = 1 \text{ kg}$ , and  $F = 3 \text{ N}$ , find the force of contact between the blocks. (b) Show that if the same force is applied to  $m_2$  rather than  $m_1$ , the force of contact between the blocks is  $2 \text{ N}$ , which is not the same as the value derived in (a). Explain.

Solution:



The applied force acting on the two bodies  $m_1$  and  $m_2$  in contact accelerate both of them. For the mass  $m_2$  to accelerate a net force is required in the same direction as its acceleration. And this force can be supplied only by  $m_1$ , which is in contact with it. This is the contact force.

Consider the two blocks together as our system.

(a) The mass of the system  $m = m_1 + m_2 = 3 \text{ kg}$

Net force on the system  $\vec{F}_{net} = \hat{x} 3 \text{ N}$

Using Newton's 2<sup>nd</sup> law  $\vec{F}_{net} = m\vec{a}$

$\therefore$  We can get  $\vec{a} = \hat{x} \frac{3}{3} = \hat{x} 1 \text{ m s}^{-2}$

Since the blocks are moving together,

the acceleration of block  $m_2$  is also  $1\text{m/s}^2$

Then, the net force on  $m_2 = m_2 \vec{a} = \hat{x} 1\text{N}$ .

$\therefore$  Contact force = 1N

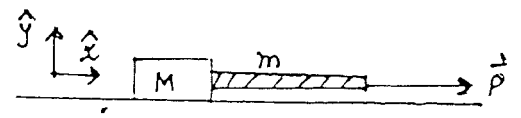
(b) Now, the same force is applied to the same system so that the acceleration of the system is still  $1\text{m/s}^2$ .

Then, the net force acting on  $m_1 = m_1 a = 2 \times 1 = 2\text{N}$

which is not the same as found in part (a). This happens because the contact force has to accelerate a larger body with the same acceleration.

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P3.10 A block of mass  $M$  is pulled along a horizontal frictionless surface by a rope of mass  $m$ , as in figure. A force  $P$  is applied to one end of the rope. (a) Find the acceleration of the block and the rope. (b) Find the force that the rope exerts on the block  $M$  in terms of  $P, M$ , and  $m$ .



Solution:

(a) Mass of the system =  $M + m$

Net force on the system  $\vec{P} = \hat{x} P$

Newton's second law gives

$$\vec{F}_{\text{net}} = m \vec{a}$$

$$\vec{P} = (M + m) \vec{a}$$

$$\vec{a} = \frac{\vec{P}}{(M + m)} = \hat{x} \frac{P}{M + m}$$

The acceleration of the block and the rope is

$$\vec{a} = \hat{x} \frac{P}{M+m}$$

(b) Net force on mass  $M$  by rope is

Net force = mass of block  $M$   $\times$  acceleration of block  $M$

$$= M \cdot \hat{x} \frac{P}{M+m}$$

$$= \hat{x} \frac{M}{M+m} P$$

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P.3.25 A string 1m long is used to whirl a  $\frac{1}{2}$  kg stone in a vertical circle at a speed of  $5 \text{ m s}^{-1}$ . (a) What is the tension in the string when the stone is at the top of the circle? (b) at the bottom?

Solution:

(a) Forces on the stone are

its weight  $\vec{W} = -\hat{y} mg$

tension  $\vec{T} = \hat{y} T$

Condition:  $a_x = 0, a_y = -v^2/R$

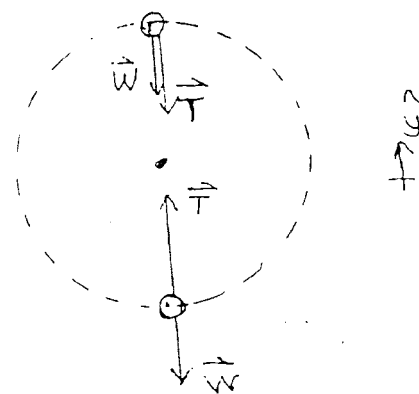
Newton's law,  $\sum F_y = ma_y$

gives  $-mg - T = -m v^2/R$

(or)  $T = (m v^2/R) - mg$

$$= (0.5 \times 5 \times 5/1) - 0.5 \times 10$$

$$= 7.5 \text{ N}$$



(b) Forces on the stone are

it weight  $\vec{W} = -\hat{y} mg$

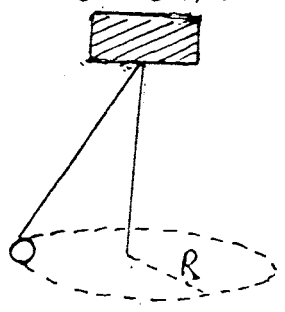
tension  $\vec{T} = +\hat{y} T$

condition:  $a_y = +\frac{v^2}{R}$

Newton's law gives  $T - mg = m\frac{v^2}{R}$

(or)  $T = mg + m\frac{v^2}{R}$   
 $= 17.5 \text{ N}$

P3.23 A conical pendulum consists of a mass of 1kg supported by a rope. It rotates in a horizontal plane with an angular velocity of  $4 \text{ rad s}^{-1}$  of radius 25cm as in figure. What is the tension in the rope?



Solution:

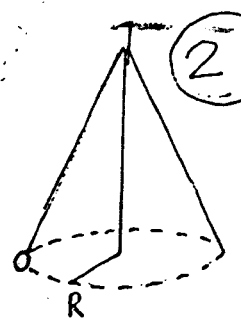
Forces acting on the pendulum are

$$\vec{W} = -\hat{y} W$$

$$\vec{T} = -\hat{x} T \sin\theta + \hat{y} T \cos\theta$$

Apply Newton's 2<sup>nd</sup> law

$$\sum F_y = ma_y = 0 \text{ (no motion in y direction)}$$



$T \cos \theta - W = 0$

$T \cos \theta = W \quad \dots \dots (1)$

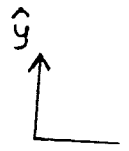
and  $\Sigma F_x = m \hat{a}_x$

(One of the components of tension,  $T \sin \theta$ , serves as centripetal forces)

$-T \sin \theta = -m a_R$

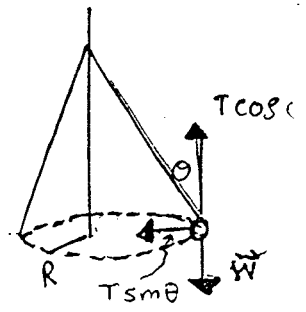
$\dots = -m R \omega^2$  (because  $a_R = R \omega^2$ )

$\dots = m T \omega^2 \quad \dots \dots (2)$



equation  $(1)^2 + (2)^2$  gives

$T^2 (\sin^2 \theta + \cos^2 \theta) = W^2 + m^2 R^2 \omega^4$



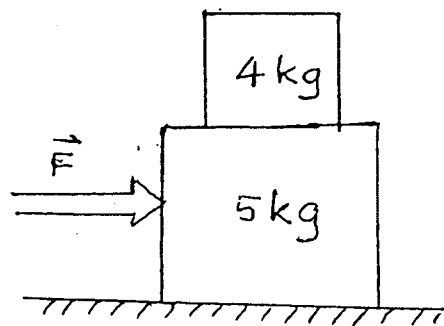
$T^2 = m^2 (g^2 + R^2 \omega^4)$

$= 1 [10^2 + (25 \times 10^2)^2 (4)^4]$

$T = 10.77 \text{ N}$

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P3.14. A 4 kg block is put on top a 5 kg block. In order to cause the top block to slip on the bottom one, a horizontal force of 12 N must be applied to the top block as in figure. Assume a frictionless table and find (a) the maximum horizontal force  $F$  which can be applied to the lower block so that the blocks will move together, and (b) the resulting acceleration of them



Solution:

The 12 N force is the limiting force of friction "f" between the blocks. It means that the maximum acceleration that can be given to the top block is

$$a = f/m_A = \frac{12}{4} = 3 \text{ m s}^{-2}$$

So that it will not slip.

In order that the blocks will move together this also is the maximum acceleration of the system. Thus the maximum force that can be given to the system is

$$(a) \vec{F} = (m_A + m_B) a = (4 + 5) \cdot 3 = 27 \text{ N}$$

(b) Of course, the resultant acceleration is  $3 \text{ m s}^{-2}$

- Chapter IV

\* Q4.4. How much work is done on a satellite during each revolution if its mass is m, its period is T, its speed is v, and its orbit is a circular orbit of radius R?

Answer: Force on the satellite is exerted by earth which directed toward the center of the earth. The displacement of the satellite is directed tangent to the circular path. Therefore, angle between force and displacement is 90°. ∴ work done by the force is equal to zero.

\* \* Q.4.6. A bullet is fired from a rifle, if the rifle were allowed to recoil freely (i.e. without being restrained by a person's shoulder) its kinetic energy as a result of recoil would be (a) equal to, (b) less than, (c) greater than that of the bullet.

Answer: Kinetic energy of gun < K.E. of bullet.

Before firing, total momentum of the system = 0. ∴ After firing, the total momentum of the system = 0

0 = m\_b v\_b - m\_G v\_G  
or m\_b v\_b = m\_G v\_G or p\_b = p\_G or p\_b^2 = p\_G^2 ----- (1)

K.E. of bullet E\_kb = 1/2 p\_b^2 / m\_b or p\_b^2 = 2 E\_kb m\_b ----- (2)

K.E. of gun E\_KG = 1/2 p\_G^2 / m\_G or p\_G^2 = 2 E\_KG m\_G ----- (3)

Substitution (2) & (3) into (1) we get

2 E\_kb m\_b = 2 E\_KG m\_G

or E\_KG / E\_kb = m\_b / m\_G

but m\_G > m\_b ∴ E\_KG < E\_kb

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Q.4.9. What does the conservation of energy principle say about the conservation of one kind of energy to another kind? Does this implies that it is always possible to convert all of one kind of energy to other kind? Explain your answer.

Answer

In conversion of one kind of energy to another the conservation of energy principle said that the total energy of the isolated system is remains constant.

It does not imply that it is always possible to convert all of one kind of energy to the other.

For example, when a body slides down on the rough inclined plane, not all of the potential energy is changed into kinetic energy; a part of it is dissipated in the form of heat.

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Q.4.11 Compare and contrast the concepts of kinetic energy and momentum. What important property does momentum have which kinetic energy lacks? Why is this property so important?

Answer: Both the kinetic energy and the momentum are properties possessed by a body by virtue of motion. Both are function of mass and velocity. But momentum is a vector quantity and kinetic energy is scalar.

An important property of momentum which kinetic energy lacks is, thus, the directional information of a motion. This property is so important for the study of motion because complete description of motion requires to include direction of motion (in addition of magnitude.).

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Q.4.13. Springs A and B are identical except that A is stiffer than B that is,  $k_A > k_B$ . On which spring is more work expected if (a) they are stretched by the same amount, (b) they are stretched by the same force?

Answer

$k_A$  = spring constant of A;  $k_B$  = spring constant of B

Given;  $k_A > k_B$

(a) They are stretched by the same amount

$$x_A = x_B = x$$

$$W_A = \frac{1}{2} k_A (x_A)^2 = \frac{1}{2} k_A x^2$$

$$W_B = \frac{1}{2} k_B (x_B)^2 = \frac{1}{2} k_B x^2$$

But  $k_A > k_B$

$\therefore W_A > W_B$

(b) When they are stretched by the same force:

$$F = kx$$

$$F_A = k_A x_A$$

$$F_B = k_B x_B$$

Since  $F_A = F_B$

$$k_A x_A = k_B x_B$$

$$\frac{x_B}{x_A} = \frac{k_A}{k_B}$$

$$\therefore \frac{W_B}{W_A} = \frac{\frac{1}{2} k_B x_B^2}{\frac{1}{2} k_A x_A^2} = \frac{k_B x_B x_B}{k_A x_A x_A} = \frac{F_B x_B}{F_A x_A} = \frac{x_B}{x_A} = \frac{k_A}{k_B}$$

Therefore  $W_B > W_A$  since  $(k_A > k_B)$ .

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Q.4.14. A man rowing a boat upstream is at rest with respect to the shore. (a) Is he doing any work? (b) If he stops rowing and moves down with the stream, is any work being done on him?

Answer:

- (a) Yes.  
The man applies a force on the water and the water is moving under him even though he is at rest with respect to the shore.
- (b) No. Although he stops rowing, he is moving with respect to the earth and so some work is being done on him by the gravitational force by the earth.

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Q.4.15

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The work done by the resultant force is always equal to the change in kinetic energy. Can it happen that the work done by one of the component forces alone will be greater than the change in kinetic energy? If so give examples.

Answer: Yes, it can happen.

Consider a block of mass which is being pulled by a string as shown in figure.

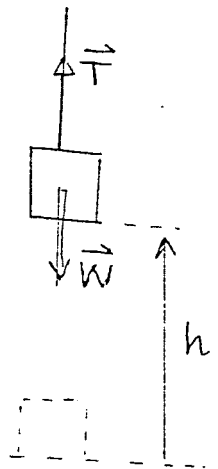
Forces acting on the block are:

(i) its weight  $\vec{W} = \hat{y}(-mg)$

(ii) tension  $\vec{T} = \hat{y}T$

$\therefore$  resultant force

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{W} + \vec{T} \\ &= \hat{y}(-mg) + \hat{y}T \\ &= \hat{y}(T - mg)\end{aligned}$$



Work done by resultant force is

$$\begin{aligned}W_{\text{net}} &= \vec{F}_{\text{net}} \cdot \vec{s} = \hat{y}(T - mg) \cdot \hat{y}h \\ &= (T - mg)h\end{aligned}$$

Work done by (a component force)  $\vec{T}$  is

$$W_T = \vec{T} \cdot \vec{s} = \hat{y}T \cdot \hat{y}h = Th$$

$$\therefore W_T > W_{\text{net}}$$

But, work done by resultant force = change in kinetic energy

$$W_{\text{net}} = \Delta E_k$$

$$\therefore W_T > \Delta E_k$$

Therefore the work done by one of the component forces can be greater than the change in kinetic energy.

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Q.4.16 Does the work done in raising a box onto a platform depend on how fast it is raised?

Answer: No, it does not depend on how fast it is raised. Work done by the force  $\vec{F}$  in taking a body from a point a to point b along a particular path is given by

$$W_{ab} = \int_a^b \vec{F} \cdot d\vec{s}$$

According to this definition, there will be a work done as long as  $\vec{F} \cdot d\vec{s}$  is not equal to zero and there is no time factor involved in this definition. So the work done is independent of time.

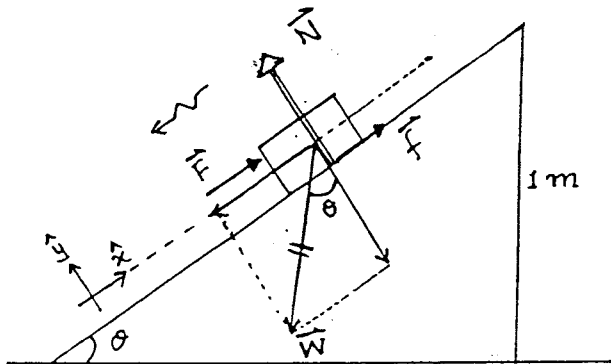
## CHAPTER IV

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## Problems

4.4 A 450 N block of ice slides down an incline 1.5 m long and 1 m high. A man pushes up on the ice parallel to the incline so that it slides down at constant speed. The coefficient of friction between the ice and the incline is 0.10. Find (a) the force exerted by the man (b) the work done by the man on the block, (c) the work done by gravity on the block, (d) the work done by the surface of the incline on the block, (e) the work done by the resultant force on the block and (f) the change in kinetic energy of the block.

Solution:



Forces exerted on the block of ice are:

(1) its weight  $\vec{W} = \hat{x}(-mg \sin \theta) + \hat{y}(-mg \cos \theta)$   
(exerted by earth)

(2) friction  $\vec{F} = \hat{x} \mu N$   
(exerted by inclined surface)

(3) Normal reaction  $\vec{N} = \hat{y} N$   
(exerted by inclined surface)

(4) force exerted by the man  $\vec{F} = \hat{x} F$

Conditions of motion:

$a_x = 0$  (moving with constant speed)

$a_y = 0$  (there is no motion in  $\hat{y}$  direction)

(29)

$$\sum F_y = m a_y \quad (\text{Newton's 2}^{\text{nd}} \text{ Law of motion})$$

$$N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$= 450 \times \cos(41.81)^\circ$$

$$= 335.4 \text{ N}$$

$$\sum F_x = m a_x$$

$$F + f - mg \sin \theta = 0$$

$$F = mg \sin \theta - f$$

$$= mg \sin \theta - \mu N$$

$$= 450 \times \sin(41.81)^\circ - 0.10 \times 335.4$$

$$= 300 - 33.54$$

$$= 266.46 \text{ N}$$

(a) The force exerted by man  $F = 266.5 \text{ N}$

(b) displacement of the ice block  $\vec{s} = \hat{x}(-L)$

where  $L = 1.5 \text{ m}$

work done by the man  $W_m = \vec{F} \cdot \vec{s}$

$$= (\hat{x} F) \cdot [\hat{x}(-L)]$$

$$= -FL$$

$$= -266.46 \times 1.5$$

$$= -399.69 \text{ J}$$

(c) work done by the normal reaction is

$$W_N = \vec{N} \cdot \vec{s}$$

$$= [\hat{y} N] [\hat{x}(-L)]$$

$$= 0$$

(d) work done by the friction is

(30)

$$\begin{aligned}W_f &= \vec{F} \cdot \vec{s} \\&= [\hat{x} F] \cdot [\hat{x} (-L)] \\&= -fL \\&= -\mu N L \\&= -0.10 \times 335.4 \times 1.5 \\&= -50.31 \text{ J}\end{aligned}$$

$\therefore$  work done by the surface of inclined is

$$\begin{aligned}W_s &= W_N + W_f \\&= 0 + (-50.31) \\&= -50.31 \text{ J}\end{aligned}$$

(e) The work done by gravity is  $W_g$

$$\begin{aligned}W_g &= \vec{W} \cdot \vec{s} \\&= [\hat{x} (-mg \sin \theta) + \hat{y} (-mg \cos \theta)] \cdot [\hat{x} (-L)] \\&= mg \sin \theta L + 0 \\&= mg \sin \theta L \\&= 450 \times 1 \quad (\sin \theta L = 1 \text{ m}) \\&= 450 \text{ J}\end{aligned}$$

(e) work done by the resultant force is

$$\begin{aligned}W &= W_m + W_g + W_s \\&= -399.69 + 450 - 50.31 \\&= 0\end{aligned}$$

(f) change in kinetic energy = work done by resultant force

$$= 0$$

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P.4.17) Show that the tension in a cord tied to a ball moving in a vertical circle at its lowest point exceeds the tension in the cord when the ball is at its highest point by six times the weight of the ball.

Solution:

At the lowest point, A

$$\vec{T} + \vec{W} = \vec{F}_{net}$$

$$T - W = \frac{mv^2}{R}$$

$$R(T - W) = mv^2$$

∴ kinetic energy at the lowest point is

$$(EK)_A = \frac{1}{2}mv^2 = \frac{R}{2}(T - W)$$

At the highest point, B

$$\vec{T}' + \vec{W} = \vec{F}_{net}$$

$$-T' - W = -\frac{mv'^2}{R}, \therefore R(T' + W) = mv'^2$$

∴ kinetic energy at the highest point is

$$(EK)_B = \frac{1}{2}mv'^2 = \frac{R}{2}(T' + W)$$

Potential energy at the lowest point is

$$(EP)_A = 0$$

Potential energy at the highest point is

$$(EP)_B = mgh = mg \cdot 2R = 2mgR$$

According to the energy conservation principle

$$(EK)_A + (EP)_A = (EK)_B + (EP)_B$$

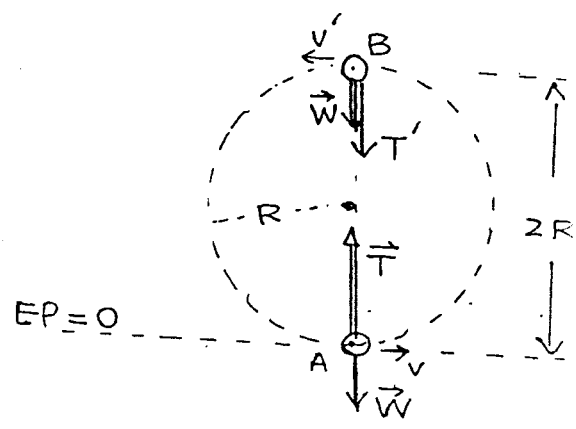
$$\frac{R}{2}(T - W) + 0 = \frac{R}{2}(T' + W) + 2mgR$$

$$T - W = T' + W + 4mg$$

$$T - T' = 2W + 4mg$$

$$T - T' = 2mg + 4mg$$

$$T - T' = 6mg$$



P. 4.20 (a) A light rod of length  $l$  has a mass  $m$  attached to its end, forming a simple pendulum. It is inverted and then released. What is its speed  $v$  at the lowest point and what is the tension in the suspension at that instant? (b) The same pendulum is next put in a horizontal position and released from rest. At what angle from the vertical will the tension in the suspension equal the weight in magnitude?

Solution:

(a) At the highest point, A

□ Kinetic energy

$$(EK)_A = \frac{1}{2} m v_0^2 = 0 \quad (\because v_0 = 0)$$

□ Potential energy

$$(EP)_A = mgh = mg \cdot 2l$$

At the lowest point, B

□ Kinetic energy

$$(EK)_B = \frac{1}{2} m v^2$$

□ Potential energy

$$(EP)_B = 0 \quad (\because h = 0)$$

Forces acting on the mass 'm' at the lowest point are

(i) its weight  $\vec{W} = \hat{r}(-mg)$

(ii) tension  $\vec{T} = \hat{r}T$

\* According to the law of conservation of energy

$$(EP)_A + (EK)_A = (EP)_B + (EK)_B$$

$$mg \cdot 2l + 0 = 0 + \frac{1}{2} m v^2$$

$$\therefore v^2 = 4gl$$

\* By Newton's second law of motion

$$\vec{F}_{\text{net}} = m\vec{a}$$

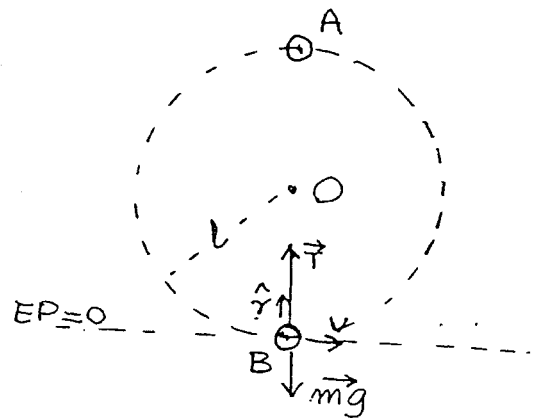
$$\hat{r}(T - mg) = \hat{r}\left(m \frac{v^2}{l}\right)$$

$$\therefore T = \frac{mv^2}{l} + mg$$

$$= \frac{m \cdot 4gl}{l} + mg$$

$$= 4mg + mg$$

$$= 5mg$$



$\hat{r}$  = unit vector directed toward centre 'O' of the circle

P.4.20 (b) Solution

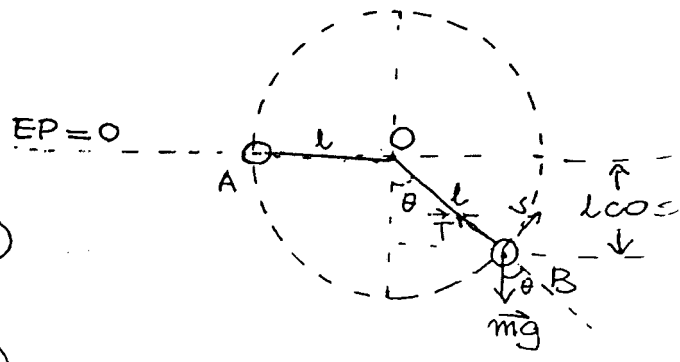
At point, A

▫ Kinetic energy

$$(EK)_A = \frac{1}{2} m v_0^2 = 0 \quad (\because v_0 = 0)$$

▫ Potential energy

$$(EP)_A = mgh = 0 \quad (\because h = 0)$$



At point, B

▫ Kinetic energy  $(EK)_B = \frac{1}{2} m v^2$

▫ Potential energy  $(EP)_B = mgh = mg(-l \cos \theta)$   
 $= -mgl \cos \theta$

According to the law of conservation of energy

$$(EK)_A + (EP)_A = (EK)_B + (EP)_B$$

$$0 + 0 = \frac{1}{2} m v^2 + (-mgl \cos \theta)$$

$$\therefore v^2 = 2gl \cos \theta \quad \leftarrow$$

\* Forces acting on the mass 'm' at point B are  
 (i) its weight  $\vec{W}$  and (ii) tension  $\vec{T}$ .

\* But, components of force in radial direction are  
 (i)  $\hat{r} T$  and (ii)  $\hat{r} (-mg \cos \theta)$

▫ By Newton's second law of motion

$$(\vec{F}_{net})_{radial} = m \vec{a}_{radial}$$

$$\hat{r} (T - mg \cos \theta) = \hat{r} \left( \frac{m v^2}{l} \right)$$

$$T - mg \cos \theta = \frac{m \cdot 2gl \cos \theta}{l}$$

$$T - mg \cos \theta = 2mg \cos \theta$$

$$T = 3mg \cos \theta$$

When  $T = mg$

$$mg = 3mg \cos \theta$$

$$\cos \theta = \frac{1}{3}$$

$$\theta = 70.53^\circ$$

P 4.22 An ideal massless spring  $S$  can be compressed 1 m by a force of 100 N. This same spring is placed at the bottom of a frictionless inclined plane which makes an angle  $\theta = 30^\circ$  with the horizontal (Fig 4.14). A 10-kg mass  $M$  is released from rest at the top of the inclined plane and is brought to rest momentarily after compressing the spring 2 m. (a) Through what distance the mass slide before coming to rest? (b) What is the speed of the mass just before it reaches the spring?

Solution:

\* For the configuration as shown in Fig. A.

◦ Kinetic energy of block

$$(EK)_A = \frac{1}{2} m v_0^2 = 0 \quad (v_0 = 0)$$

◦ Gravitational potential energy of the block

$$(EPG)_A = mgh = 0 \quad (\because h = 0)$$

◦ Elastic potential energy of the spring

$$(EPE)_A = \frac{1}{2} k x^2 = 0 \quad (\because x = 0)$$

Total mechanical energy

$$\begin{aligned} E_A &= (EK)_A + (EPG)_A + (EPE)_A \\ &= 0 + 0 + 0 \\ &= 0 \end{aligned}$$

\* For the configuration as shown in Fig. B.

◦ Kinetic energy of block

$$(EK)_B = \frac{1}{2} m v^2$$

◦ Gravitational potential energy of the block

$$(EPG)_B = -mgh_1$$

◦ Elastic potential energy of the spring

$$(EPE)_B = \frac{1}{2} k x^2 = 0 \quad (\because x = 0)$$

Total mechanical energy

$$E_B = -mgh_1 + \frac{1}{2} m v^2$$

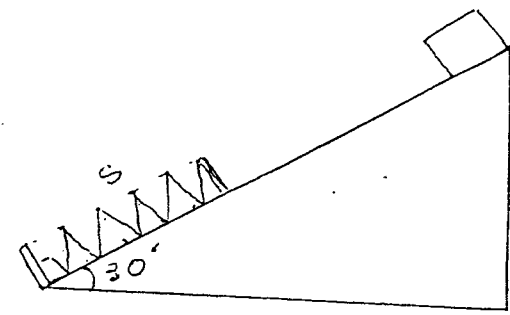


Fig 4.14

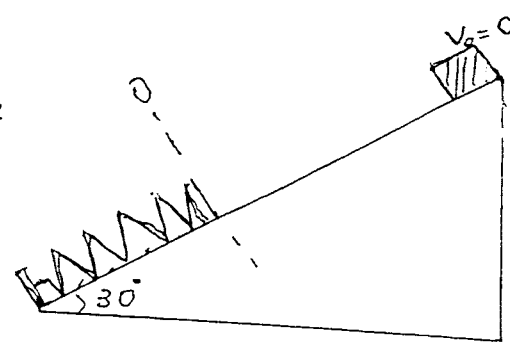


Fig. A

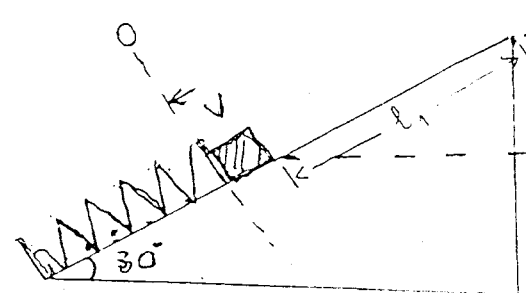


Fig. B

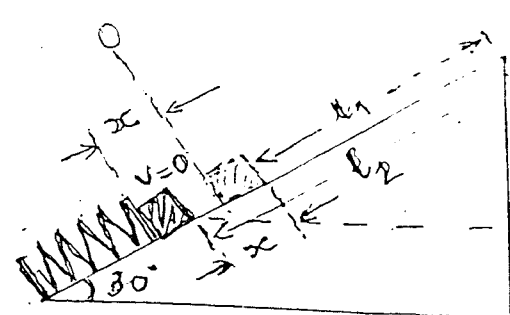


Fig. C  $l_2 = l_1 + x$

\* For the configuration as shown in Fig. C

◦ Kinetic energy of block

$$(EK)_C = 0 \quad (v=0)$$

◦ Gravitational potential energy of block

$$(EPG)_C = mgh = -mgh_2$$

◦ Elastic potential energy of spring

$$(EFE)_C = \frac{1}{2} kx^2$$

◦ Total mechanical energy  $E_C = \frac{1}{2} kx^2 - mgh_2$

\* Since there is no frictional forces, total mechanical energy is conserved.

$$\text{spring constant } k = \frac{F}{x} = \frac{100 \text{ N}}{1 \text{ m}}$$

$$= 100 \text{ N m}^{-1}$$

(a)

$$E_A = E_C \quad (\text{Total mechanical energy conserve})$$

$$0 = \frac{1}{2} kx^2 - mgh_2$$

$$h_2 = \frac{kx^2}{2mg}$$

$$l_2 \sin \theta = \frac{kx^2}{2mg}$$

$$l_2 = \frac{kx^2}{2mg \sin \theta} = \frac{100 \times 2^2}{2 \times 10 \times 10 \times \sin 30^\circ} = 4 \text{ m}$$

(b)

$$l_2 - l_1 = x \quad \therefore l_1 = l_2 - x = 4 - 2 = 2 \text{ m}$$

$$E_A = E_B \quad (\text{Total mechanical energy conserve})$$

$$0 = -mgh_1 + \frac{1}{2} mv^2$$

$$v^2 = 2gh_1$$

$$= 2g l_1 \sin 30^\circ$$

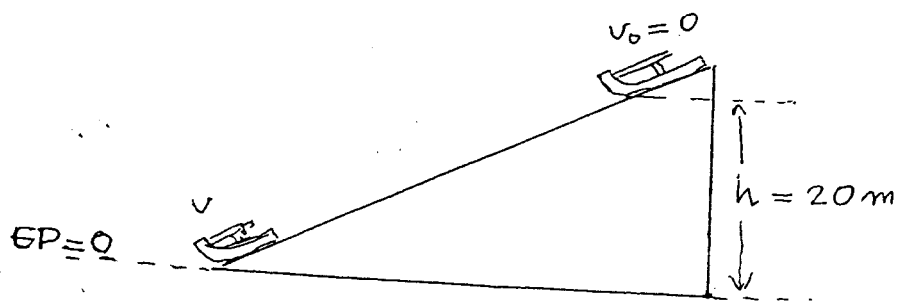
$$= 2 \times 10 \times 2 \sin 30^\circ$$

$$= 20$$

$$v = 4.47 \text{ m s}^{-1}$$

\*\*\*  
 P.4.26. A sled with a mass of 29 kg slides on a hill, starting at an altitude of 20 m. The sled starts from rest and has velocity of  $16 \text{ m s}^{-1}$  when it reaches the bottom of the slope. Calculate the loss of energy due to friction.

Solution:



$$\text{Initial kinetic energy } (EK)_i = \frac{1}{2} m v_0^2 = 0$$

$$\begin{aligned} \text{Initial potential energy } (EP)_i &= mgh \\ &= 29 \times 10 \times 20 \\ &= 5800 \text{ J} \end{aligned}$$

$\therefore$  Initial total mechanical energy is

$$\begin{aligned} E_i &= (EK)_i + (EP)_i \\ &= 0 + 5800 = 5800 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Final kinetic energy } (EK)_f &= \frac{1}{2} m v^2 \\ &= \frac{1}{2} \times 29 \times (16)^2 \\ &= 3712 \text{ J} \end{aligned}$$

$$\text{Final potential energy } (EP)_f = mgh = 0 \quad (\because h=0)$$

$\therefore$  Final total mechanical energy is

$$\begin{aligned} E_f &= (EK)_f + (EP)_f \\ &= 3712 + 0 = 3712 \text{ J} \end{aligned}$$

$$\begin{aligned} \therefore \text{Loss of energy due to friction} &= E_f - E_i \\ &= 3712 - 5800 \\ &= -2088 \text{ J} \end{aligned}$$