

Esc – 01011
ENGINEERING SCIENCE

Part (II)

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3. Derived Quantities and Units

Derived quantity - Composed of basic quantity.

Their units are obtained by using the product or quotient of basic units.

Examples:

Quantity	Expressed in Basic Unit	Special Name
Volume	m^3	
Velocity	$m\ s^{-1}$	
Force	$kg\ m\ s^{-2}$	newton (N)
Work	$kg\ m^2\ s^{-2}$	joule (J)

The SEVEN basic units in SI units are defined as follow:

- (i). The **meter** (m) is the unit of length, defined in terms of the **wavelength of light from the Krypton-86 atom.**
- (ii). The **kilogram** (kg) is the unit of mass, defined by an international prototype at Sevres, France.
- (iii). The **second** (s) is the unit of time, defined in terms of **the frequency of light from the cesium-133 atom.**
- (iv). The **ampere** (A) is the unit of current, defined in terms of the force between two parallel conductors (carrying equal current) one meter apart.
- (v). The **Kelvin** (K) is the unit of thermodynamic temperature.
- (vi). The **mole** (mol) is the unit of amount of a substance.
- (vii). The **candela** (cd) is the unit of luminous intensity.

5. Very Large and Small Numbers

For very large or very small numbers, we sometimes use a shorthand way of writing them, by counting the number of zeros. These abbreviations are known as **prefixes**.

Prefixes for power of 10

Power of 10	Prefix	Abbreviation	Pronunciation
10^{-18}	atto-	a	at-toe
10^{-15}	femto-	f	fem-toe
10^{-12}	pico-	p	pee-koe
10^{-9}	nano-	n	nan-oe
10^{-6}	micro-	μ	my-kroe
10^{-3}	milli-	m	mil-i
10^{-2}	centi-	c	cen-ti
10^3	kilo-	k	kil-oe
10^6	mega-	M	meg-a
10^9	giga-	G	jig-a
10^{12}	tera-	T	ter-a
10^{15}	peta-	P	pet-a
10^{18}	exa-	E	ex-a

4. Dimensions

Dimension - **Power** which is needed to put on **basic quantities**.

Dimensions of the fundamental quantities such as *mass*, *length* and *time* are denoted by [M], [L] and [T] respectively. Note: [], **square bracket**, is used to describe dimension of respective basic quantity.

Each basic unit has an associated dimension.

i.e., in mechanics, dimension of a physical quantity = $[M^x L^y T^z]$ where x, y, and z may be positive or negative. The power x, y, and z stand for dimension of respective physical quantity.

Physical quantities can be categorized, as shown in below, according to their dimensions.

	variable	constant
Quantity with dimension	v, F, w, P, E _p , E _k ,...	g, G, ε ₀ ,...
Dimensionless quantity	θ, ε _r , μ _r ,	2, 3, π, ...

Some examples are ~

$$\text{velocity} = \frac{\text{length}}{\text{time}} = [L T^{-1}]$$

$$\text{acceleration} = \frac{\text{length}}{\text{time} \times \text{time}} = [L T^{-2}]$$

$$\text{force} = \text{mass} \times \text{acceleration} = [M L T^{-2}]$$

$$\text{pressure} = \frac{\text{force}}{\text{area}} = [M L^{-1} T^{-2}]$$

$$\text{work} = \text{force} \times \text{distance} = [M L^2 T^{-2}]$$

$$\text{kinetic energy} = \frac{1}{2} \times \text{mass} \times (\text{velocity})^2 = [M L^2 T^{-2}]$$

The quantities having different dimensions **can not be added** or **subtracted** but **can be multiplied** or **divided** by one another. When we have two quantities “A” and “B” having dimensions of

$$[M^{x1} L^{y1} T^{z1}] \text{ and } [M^{x2} L^{y2} T^{z2}] \text{ respectively, then}$$

$$[A B] = [M^{x_1+x_2} L^{y_1+y_2} T^{z_1+z_2}]$$

$$[A/B] = [M^{x_1-x_2} L^{y_1-y_2} T^{z_1-z_2}]$$

Some physical quantities such as relative density are dimensionless.

A **dimensional equation** is an equation between various physical quantities expressed in dimensional notation. They can be used for **conversion of units**, **checking the equations**, and **obtaining the form of a physical quantity**. These procedures are collectively known as dimensional analysis.

5. Dimensionless or Unitless Quantities

These are quantities which are **ratios** of quantities having the **same dimensions or units**. Examples include relative density, angles, etc.

6. Homogeneity of Physical Equations

A **physically correct equation** must have the same dimensions or units on both the right-hand side (RHS) and the left-hand side (LHS) of the equation.

Example 1.1

A student claims that the period of a simple pendulum **T** is related to its length **l** , mass of the bob **m** , and acceleration due to gravity **g** by the

following formula : $T = k \sqrt{\frac{m l}{g}}$ where k is a dimensionless constant.

Is the student right?

Solution

LHS : Units of T = s

$$\text{RHS : Units of } k \sqrt{\frac{m \ell}{g}} = \left(\frac{\text{kg m}}{\text{m s}^{-2}} \right)^{\frac{1}{2}} = (\text{kg s}^{-2})^{\frac{1}{2}}$$

The quantities on both sides of the equation do not have the same unit. The equation is inhomogeneous and therefore physically wrong.

Alternative solution

Dimensions of T : [T]

$$\text{Dimensions of } k \sqrt{\frac{m \ell}{g}} : \left[\frac{\text{M L}}{\text{L T}^{-2}} \right]^{\frac{1}{2}} = [\text{M T}^2]^{\frac{1}{2}}$$

The equation is **dimensionally inconsistent** and therefore physically wrong.

Example 1.2

A second student claims the following formula for the period of a simple pendulum $T = k \sqrt{\frac{\ell}{g}}$ where the symbols have their usual meanings.

Comment on his claim.

Solution

LHS : Units = s

$$\text{RHS : Units} = \left(\frac{\text{m}}{\text{m s}^{-2}} \right)^{\frac{1}{2}} = \text{s}$$

The equation is **homogeneous**. It is also said to be **dimensionally consistent**.

Note A rigorous mathematical treatment yields the value of k to be 2π . The correct physical formula for the period is hence $T = 2\pi \sqrt{\frac{\ell}{g}}$. If

instead, the period is written as $T = 3\pi \sqrt{\frac{\ell}{g}}$, it can be shown that the equation is still dimensionally consistent but it is physically wrong. The conclusion is :

*Equations which are governed by laws of physics **must be** dimensionally consistent. However, dimensionally consistent equations **need not be** governed by laws of physics.*

7. Unit Conversion

Let “Q” be a physical quantity with a dimension $[M^X L^Y T^Z]$.

Suppose that the numerical value of Q in a system of units of $M_1 L_1 T_1$ be n_1 . Then in this system.

$$Q = n_1 [M_1^X L_1^Y T_1^Z]$$

If the numerical value of Q in some other system units of $M_2 L_2 T_2$ were n_2 , that is

$$Q = n_2 [M_2^X L_2^Y T_2^Z]$$

Therefore $n_1 [M_1^X L_1^Y T_1^Z] = n_2 [M_2^X L_2^Y T_2^Z]$

This relation gives the conversion factor useful for changing the numerical value of the quantity Q from one unit to the other when the ratios of the units of the two systems are known.

Example 1.2 Convert $g = 32.17 \text{ ft s}^{-2}$ into SI units.

Solution: The quantity “g” being the acceleration due to gravity has a dimension of $[\text{L T}^{-2}]$.

Moreover,
$$\left[\frac{\text{L}_1}{\text{L}_2} \right] = \frac{1 \text{ ft}}{1 \text{ m}} = \frac{0.3048 \text{ m}}{1 \text{ m}} = 0.3048$$

and
$$\left[\frac{\text{T}_1}{\text{T}_2} \right] = \frac{1 \text{ s}}{1 \text{ s}} = 1$$
 since the numerical value of “g” in British

Engineering units is 32.17, we denote

$$n_1 = 32.17$$

thus, we can use

$$\begin{aligned} n_2 &= n_1 \left[\frac{\text{L}_1}{\text{L}_2} \right] \left[\frac{\text{T}_1}{\text{T}_2} \right]^{-2} \\ &= 32.17 [0.3048] [1]^{-2} \\ &= 9.81 \end{aligned}$$

Thus,
$$g = 9.8 \text{ m s}^{-2}$$

QUESTIONS

1. What do you understand by the term “dimension of a physical quantity”?
2. Write down the dimensional notations for the following quantities:
 - (a) momentum, (b) the constant of gravitation G, (c) potential energy, (d) kinetic energy, (e) torque or moment of force.
3. State the principle of dimensional homogeneity.

4. What are the fundamental quantities? How many are there, and what are they?
5. What are the derived quantities?
6. In the statement “The mass of a block of wood is 2 kilogram”, what does kilogram stands for and what does 2 stands for?
7. Is mass a fundamental physical quantity (or) derived quantity? Give reason.
8. Is force a fundamental physical quantity? Give reasons.

PROBLEMS

1. Change 1 lb force into newtons.
2. Express the standard atmospheric pressure of 1.013×10^6 dynes cm^{-2} into SI and FPS units.
3. Test by the method of dimensions, the correctness of the following equations :

$$v = v_0 + at$$

$$s = v_0t + \frac{1}{2}at^2$$

Where s = distance traveled, v_0 = initial velocity

v = final velocity, a = acceleration, and t = time

CHAPTER II

KINEMATICS OF A PARTICLE

2.1 Kinematics : A branch of mechanics which deals with the *description* of the motion of objects.

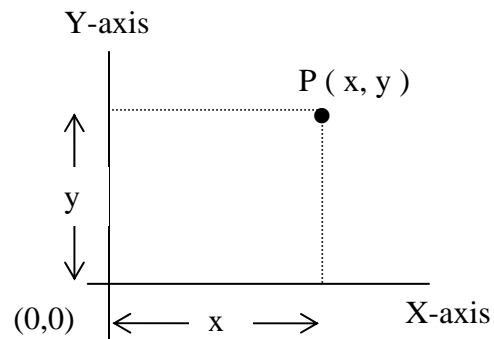
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Motional quantities

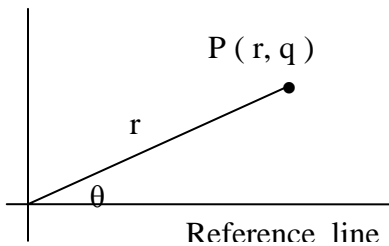
To describe the *complete information* of a motion, of an object, (i) *position (location)* (ii) *velocity* and (iii) *acceleration* of that object have to be mentioned.

Mathematical Note:

- **Rectangular coordinate system** (Cartesian System)
for two dimension



- **Polar coordinate system**



If reference line of **polar** coordinate system lies on **x-axis of Cartesian** system.

$$x = r \cos \theta \quad y = r \sin \theta \quad \text{----- (2.1)}$$

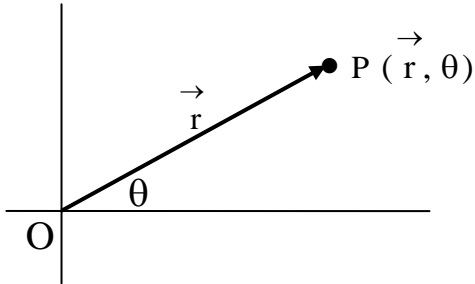
$$r = \sqrt{x^2 + y^2} \quad \tan \theta = \frac{y}{x} \quad \text{----- (2.2)}$$

2.2 Position and Frame of Reference

To describe the location of an object, a proper coordinate system has to be used. Such a coordinate system is known a *frame of reference*.

Position Description of location of an object with respect to a reference - frame.

Position Vector: Directed straight line segment, which starts from some reference point 'O' and ends at the location of the object 'P' that wants to be mentioned.



$$\vec{r} = \vec{OP} = \text{position vector of P}$$

vtm02vd1:

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mathematical note

Vector : A quantity which requires both **magnitude** and **direction** to describe its complete information.

e.g - position vector, displacement, velocity, acceleration, force, etc...

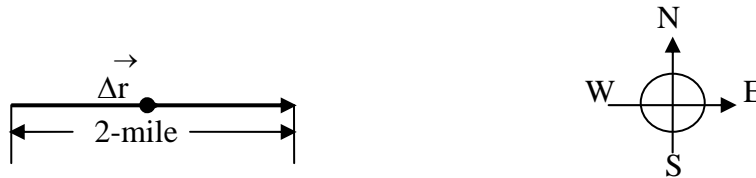
: A vector can be represented by arrow-headed line segment.

: **Direction of arrow represents direction of vector. Length of arrow is proportional to the magnitude of vector.**

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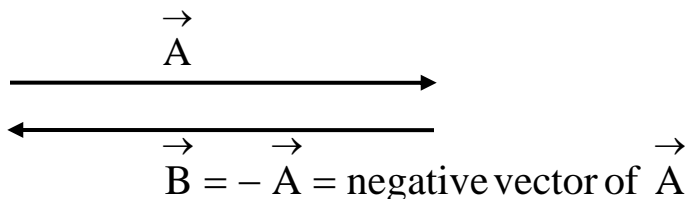
e.g. An object moves 2-miles towards east of its original position.

Such displacement ($\vec{\Delta r}$) can be described as follow ~



Negative Vector of a Vector

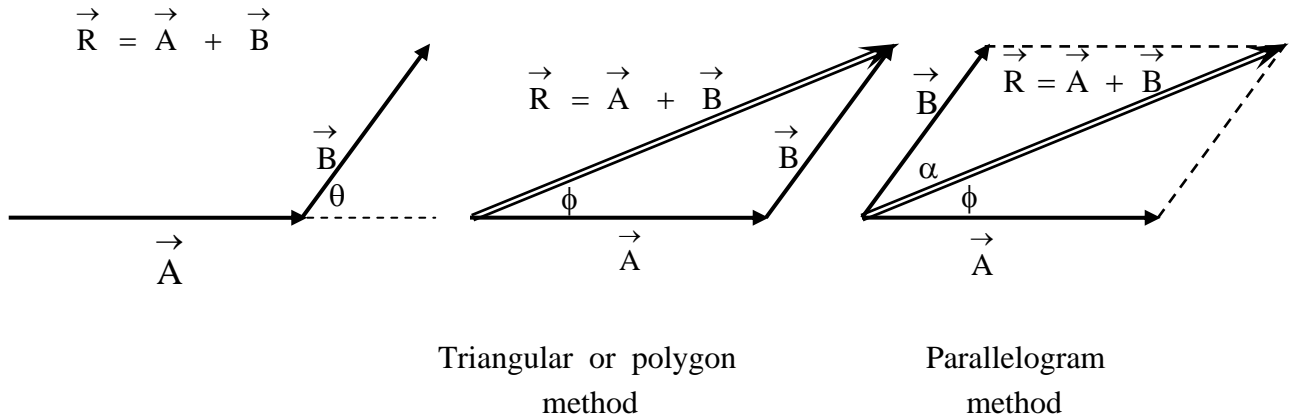
A vector having **same magnitude** with **opposite direction** to the given vector.



vta04an1:

vtm04an1:

Addition of Vectors



vtm04an2:

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

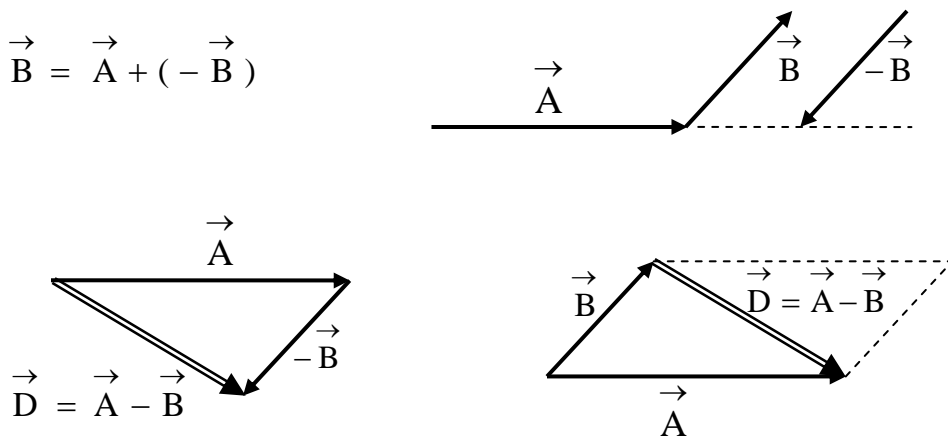
$$\phi = \tan^{-1} \left[\frac{B \sin \theta}{A + B \cos \theta} \right]$$

$$\alpha = \tan^{-1} \left[\frac{A \sin \theta}{B + A \cos \theta} \right]$$

$$\frac{\sin \alpha}{A} = \frac{\sin \phi}{B} = \frac{\sin \theta}{R}$$

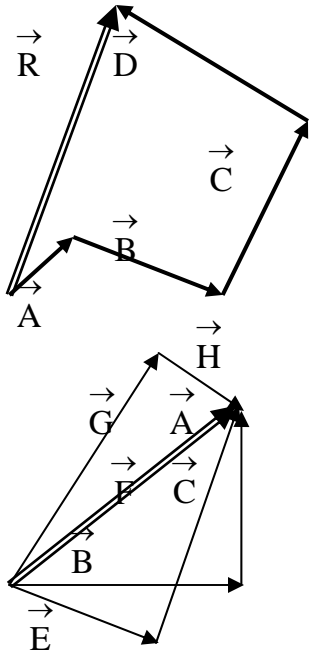
Subtraction of Vectors

$$\vec{D} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



vta04an2:

Resolution of a Vector



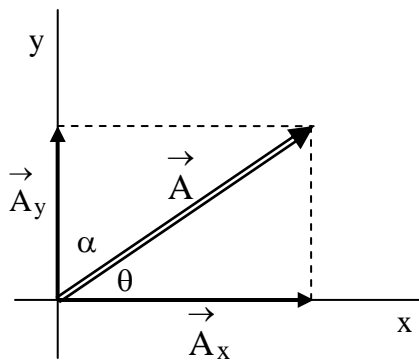
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

\vec{R} = Resultant vector

$\vec{A}, \vec{B}, \vec{C}, \vec{D}$ = component vector of \vec{R}

$$\vec{A} = \vec{B} + \vec{C} = \vec{E} + \vec{F} = \vec{G} + \vec{H}$$

\vec{B} and \vec{C} , \vec{E} and \vec{F} , \vec{G} and \vec{H} are rectangular components of \vec{A} .



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$A_x = A \cos \theta = A \sin \alpha$$

$$A_y = A \sin \theta = A \cos \alpha$$

vtm05an2:

Unit Vector : Unit vector of a vector is having **unit magnitude** and same direction of the given vector.

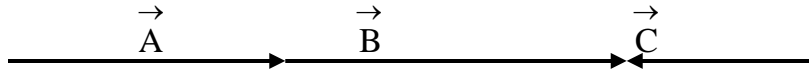
$$\hat{n} \quad \vec{A} \quad \text{unit vector of } \vec{A}, \hat{n} = \frac{\vec{A}}{A}$$

vta05an1;

vta05an2:

Signed Scalar

It can be used for vectors those lying on a straight line.



$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

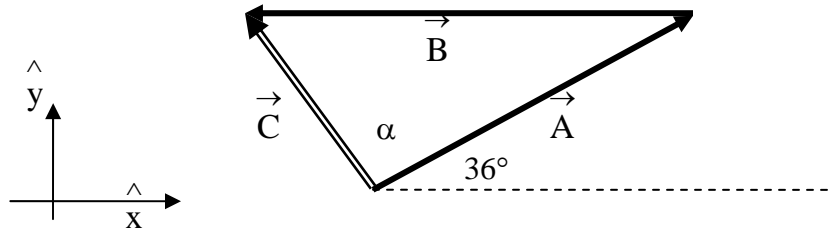
In signed scalar form, $R = A + B - C$

Example 2.1

Given two vectors : A is 6 units long and makes an angle of 36° with the positive x-axis : B is 7 units long and is in the direction of negative x-axis. Find the sum of these two vectors.

Solution :

The given two vectors A and B are drawn to scale with respect to a set of rectangular coordinate axes (x and y), to form sides of a triangle with B starting from the end of A. The third side C, then, gives the resultant vector directed from the start of A to the end of B as shown in the figure below.



Then, applying cosine rule, we have

$$\begin{aligned} C^2 &= A^2 + B^2 - 2AB \cos 36^\circ \\ &= (6)^2 + (7)^2 - (2)(6)(7) \cos 36^\circ \end{aligned}$$

which gives $C = 4.128$ units.

To find the angle between \vec{C} and \vec{A} , we have to use sine rule to get

$$\begin{aligned} \sin \alpha &= \frac{B \sin 36^\circ}{C} \\ \sin \alpha &= \frac{(7)(\sin 36^\circ)}{(4.128)} \end{aligned}$$

which gives $\alpha = 85^\circ 22'$

Therefore, \vec{C} is 4.128 units long and in the direction that makes an angle of $(36^\circ + 85^\circ 22')$ or $121^\circ 22'$ with the positive x-axis.

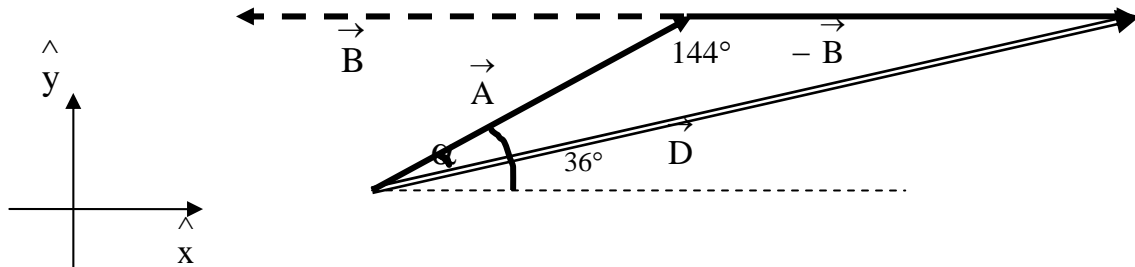
Example 2.2

Find the difference vector $\vec{D} = \vec{A} - \vec{B}$ of Example 2.1.

Solution

In this problem, the negative vector of \vec{B} must be added \vec{A} as we want to subtract \vec{B} from \vec{A} .

To find the difference vector $\vec{D} = \vec{A} - \vec{B}$, the negative vector of \vec{B} is drawn to scale with respect to a set of rectangular coordinate (x and y) axes, starting from the end of \vec{A} to form sides of a triangle taken in order. The third side \vec{D} , then, gives the difference vector directed from the start of \vec{A} to the end of $-\vec{B}$.



Then, applying the cosine rule, we have

$$\begin{aligned} D^2 &= A^2 + B^2 - 2AB \cos 144^\circ \\ &= (6)^2 + (7)^2 - (2)(6)(7) \cos 144^\circ \end{aligned}$$

which gives $D = 12.37$ units

To find the angle between \vec{D} and \vec{A} we have to use the sine rule to get

$$\begin{aligned} \sin \alpha &= \frac{|-\vec{B}| \sin 144^\circ}{D} \\ \sin \alpha &= \frac{(7)(\sin 144^\circ)}{(12.37)} \end{aligned}$$

which gives $\alpha = 10^\circ 26'$.

We have indicated $|\vec{-B}|$ instead of just B even though they are the same numerically as we have used the formula of sine rule to find the direction of the resultant of the sum of \vec{A} and $\vec{-B}$.

Therefore, \vec{D} is 12.37 units long and in the direction that makes an angle of $(36^\circ - 19^\circ 26')$ or $16^\circ 34'$ with the positive x-axis.

Example 2.3

Find by the method of components, the resultant of a vector of magnitude 12 that is at an angle 30° from the x-axis and another of magnitude 8 that is at an angle of 100° from the x-axis.

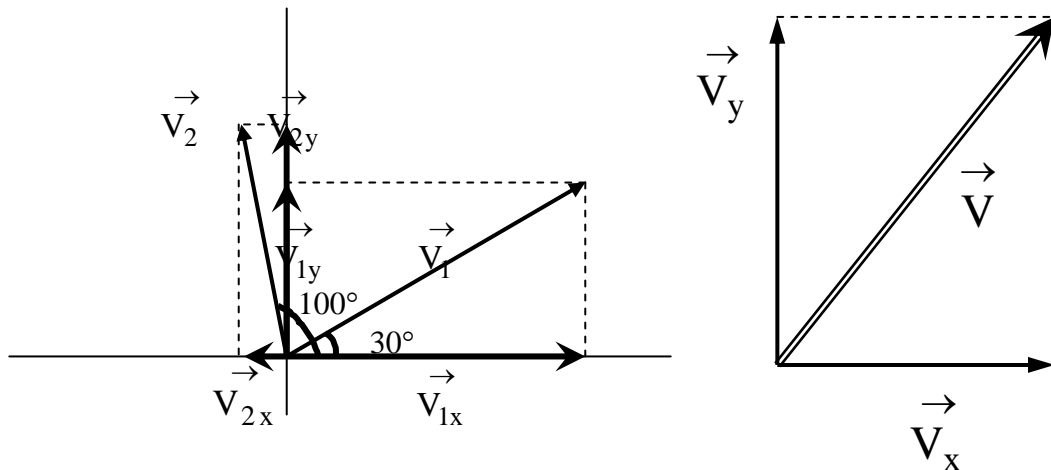
Solution :

Given two vectors are drawn to scale with respect to a set of coordinate axes (x and y), connecting their ends the next step is to resolve each of the vectors into x-and y- components.

Calling these two vectors \vec{V}_1 and \vec{V}_2 , respectively, their rectangular components can be expressed as

$$\vec{V}_1 = \vec{V}_{1x} + \vec{V}_{1y} = \hat{x} V_{1x} + \hat{y} V_{1y}$$

$$\vec{V}_2 = \vec{V}_{2x} + \vec{V}_{2y} = \hat{x} V_{2x} + \hat{y} V_{2y}$$



We can see from the above figure that the signed scalars of x-components are

$$V_{1x} = V_1 \cos 30^\circ = 12 \cos 30^\circ = 10.4,$$

$$V_{2x} = V_2 \cos 100^\circ = 8 \cos 100^\circ = -1.4,$$

and then the signed scalar of y-component are

$$V_{1y} = V_1 \sin 30^\circ = 12 \sin 30^\circ = 6.0,$$

$$V_{2y} = V_2 \sin 100^\circ = 8 \sin 100^\circ = 7.9,$$

the resultant signed scalar of x-component is

$$V_x = V_{1x} + V_{2x} = 9.0$$

and that of y-component is

$$V_y = V_{1y} + V_{2y} = 13.9,$$

so that
$$\vec{V}_1 = \hat{x} 9.0 + \hat{y} 13.9$$

To find the magnitude of the resultant vector of \vec{V}_1 and \vec{V}_2 , we have to use the equation of the form $V = \sqrt{V_x^2 + V_y^2}$ which gives, $V = 16.5$ its direction can be obtained from the equation

$$\theta = \tan^{-1} \left[\frac{V_y}{V_x} \right]$$

we get, $\theta = 57^\circ$.

Therefore, the resultant of given two vectors is 16.5 long and in the direction that makes an angle of 57° with the positive x-axis.

2.3 Motional Quantities

2.3.1 Path (trajectory)

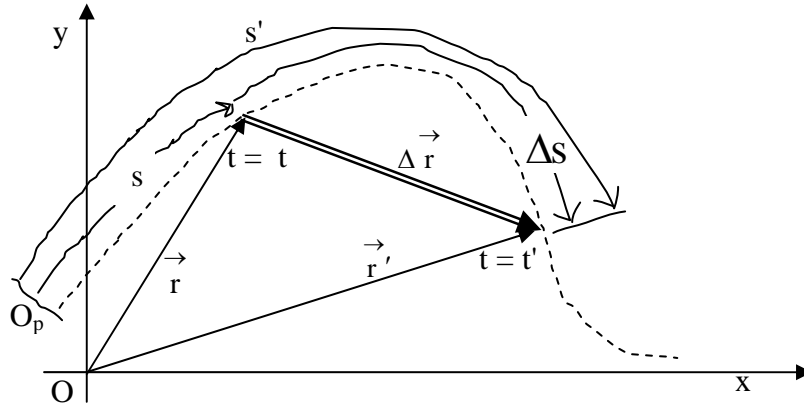
Locus of points representing the positions of the particle throughout its motion.

2.3.2 Distance (Distance covered)

Length measured along the path.

$$\Delta s = s' - s$$

It is a **scalar** quantity and its SI unit is “**meter**”



Distance and displacement

2.3.3 Displacement (Change in position)

Directed straight line segment from initial position to final position.

$$\vec{\Delta r} = \vec{r}' - \vec{r}$$

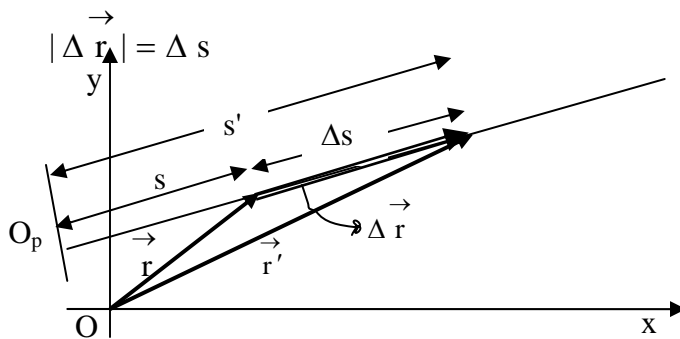
It may be also expressed in terms of rectangular components

$$\text{(i.e.)} \quad \vec{\Delta r} = \hat{x} \Delta x + \hat{y} \Delta y$$

It is a **vector** quantity and its SI unit is "**meter**".

note In general $|\vec{\Delta r}| \neq \Delta s$

If motion is a rectilinear and without back tracking.



2.4 Average speed The ratio of distance covered to time elapsed (or) the average rate of distance covered.

$$\text{average speed} = \frac{\text{distance covered}}{\text{time elapsed}}, \quad \bar{v} = \frac{\Delta s}{\Delta t}$$

Instantaneous speed : The rate of change of distance covered. (or) The limiting value of average speed.

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Speed is a scalar quantity and

its SI unit is m s^{-1} .

2.5. Average velocity

The ratio of displacement (change in position) to time elapsed. (or) The average rate of displacement.

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}}, \quad \vec{v}_{\text{av}} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity The rate of change of position (or). The rate of displacement (or). The limiting value of average velocity.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}, \text{ Velocity is a vector quantity and its SI unit is } \text{m s}^{-1}.$$

note: Direction of instantaneous velocity is *always tangential* to the path. Directions of average and instantaneous velocity are the same only for rectilinear motion without back tracking.

- Magnitude of instantaneous velocity is *always* the same as instantaneous speed.
- For a motion with 'constant' velocity, it must be a *straight line motion* and in such cases average velocity and instantaneous velocity are the same.
- Normally 'velocity' means 'instantaneous velocity'.

2.6 Average acceleration The ratio of change of velocity to time elapsed.

$$\text{Average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}, \quad \vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v} - \vec{v}_0}{t - t_0}$$

where, \vec{v} is velocity at 't' and \vec{v}_0 is velocity at 't₀'.

Instantaneous acceleration

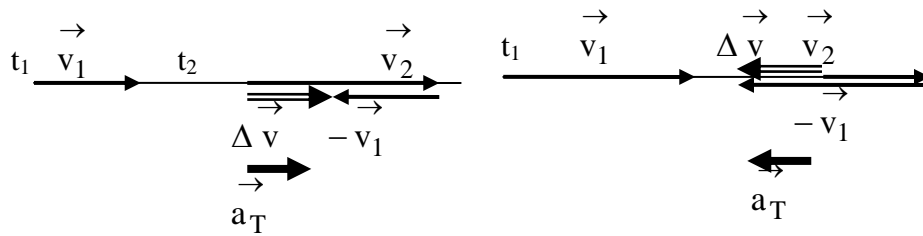
The ratio of change of velocity (or) . The limiting value of average acceleration.

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Acceleration is a **vector quantity** and its SI unit is **m s⁻²**.

note Direction of acceleration is *always* the same as that of **change of velocity**.

2.6.1 Tangential Acceleration The rate of change of **magnitude** of velocity.



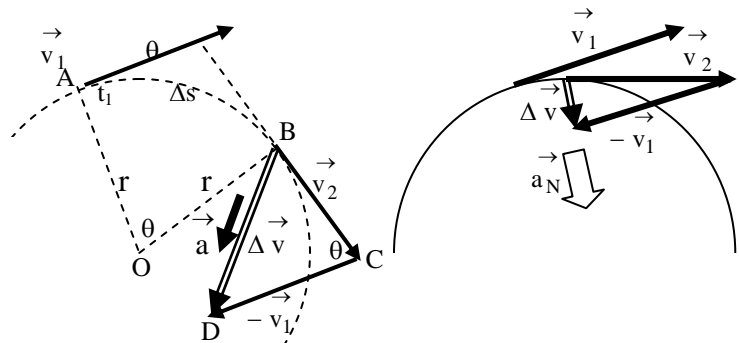
$$\vec{a}_T = \frac{v_2 - v_1}{t_2 - t_1}, \quad \text{in signed scalar form,} \quad a_T = \frac{v_2 - v_1}{t_2 - t_1}.$$

note If tangential acceleration is positive direction of a_T and that of motion are the same and motion is an accelerated motion. [In this case, speed will be increased]. If tangential acceleration is negative, direction of a_T and that of motion are opposite and motion is a decelerated motion or motion under negative acceleration. [In this case speed will be decreased].

2.6.2 Normal Acceleration

The rate of change of **direction** of velocity.

$$V = |\vec{V}_1| = |\vec{V}_2|$$



If θ is small enough (i.e. $\Delta t \rightarrow 0$)

ΔABO and ΔBCD are similar triangles

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \frac{\Delta s}{\Delta t}$$

since $\Delta t \rightarrow 0$

$$\frac{dv}{dt} = \frac{v}{r} v = \frac{v^2}{r}$$

$$a = a_R = a_C = a_N = \frac{v^2}{r} = r \omega^2 \quad [v = r \omega]$$

note Since \vec{a} is perpendicular to the path ' \vec{v} ' is known as normal acceleration ' a_N '.

Since \vec{a} is directed to the centre of curvature of path, ' \vec{a} ' is known as centripetal acceleration ' a_C '.

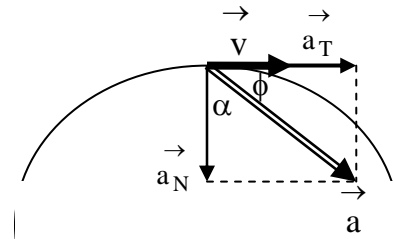
Since \vec{a} is directed along the radius of curvature, ' \vec{a} ' is known as radial acceleration ' a_N '.

In general,

$$\vec{a} = \vec{a}_T + \vec{a}_N$$

$$a = \sqrt{a_T^2 + a_N^2}$$

$$\phi = \tan^{-1} \left(\frac{a_N}{a_T} \right) \quad (\text{or}) \quad \alpha = \tan^{-1} \left(\frac{a_T}{a_N} \right)$$



note

$$\frac{d|\vec{v}|}{dt} \neq \left| \frac{d\vec{v}}{dt} \right|$$

The first is the rate of change of speed ; it is zero whenever a particle moves with constant speed, even when its direction of motion changes. The second is the magnitude of the acceleration vector ; it is zero only when the

particle's acceleration is zero, that is, when the particle moves in a straight line with constant velocity.

Example 2.4

A particle moves along the x-axis in such a way that its position at any instant is given by $x = 5t^2 + 1$, where x is in meters and t in seconds. Compute its average speed for the interval between (a) 2 s and 3 s, (b) 2 s and 2.1 s, (c) 2 s and 2.001 s, and (d) also find its instantaneous speed at 2 s.

Solution:

t_0 will be taken to be 2 s as it is common for the entire problem.

Using $x = 5t^2 + 1$, we have $x_0 = 5(2)^2 + 1 = 21$ m

Therefore, for each case $\Delta x = x - x_0 = (5t^2 + 1) - (21) = 5t^2 - 20$

$$\Delta t = t - t_0 = t - 2$$

and
$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{5t^2 - 20}{t - 2}$$

(a) For $t = 3$ s,
$$\bar{v} = \frac{5(3)^2 - 20}{3 - 2} = 25 \text{ m s}^{-1}$$

(b) For $t = 2.1$ s,
$$\bar{v} = \frac{5(2.1)^2 - 20}{2.1 - 2} = 20.5 \text{ m s}^{-1}$$

(c) For $t = 2.001$ s,
$$\bar{v} = \frac{5(2.001)^2 - 20}{2.001 - 2} = 20.005 \text{ m s}^{-1}$$

It can be noticed that as Δt becomes smaller, the average speed approaches the value of 20 m s^{-1} .

(d) We may thus expect that 20 m s^{-1} is the instantaneous speed at 2 s. In fact,

$$v = \frac{dx}{dt} = \frac{d}{dt}(5t^2 + 1) = 10t$$

When we set $t = 2$ s, then we obtain, $v = 20 \text{ m s}^{-1}$.

note

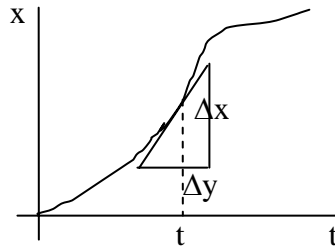
Graphical Representation

Key-interest - Nature of the graph

- Slope

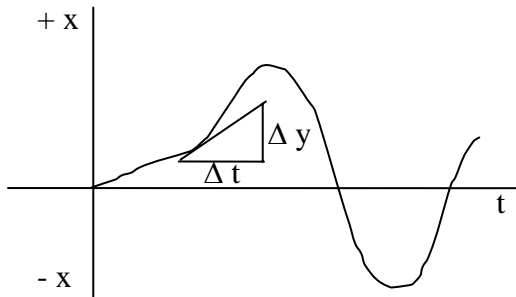
- Point of Intersection
- Area under the curve

Distance vs. Time Graph (for 1-D motion)



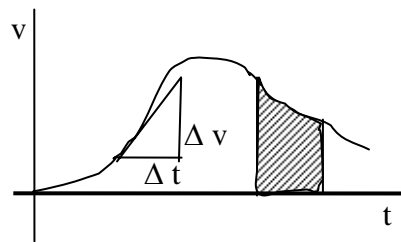
$$\text{slope} = \text{speed} = \frac{\Delta x}{\Delta t}$$

Displacement vs. Time Graph (for 1-D motion)



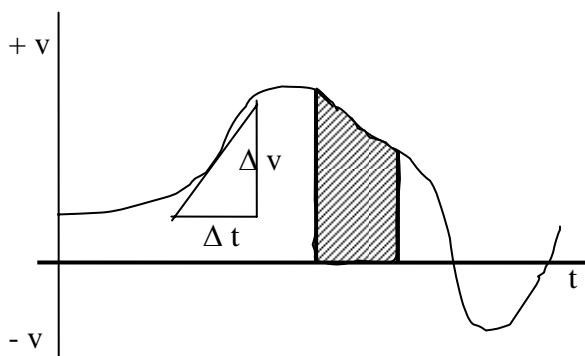
$$\text{slope} = \text{velocity}$$

Speed vs. Time Graph (for 1-D motion)



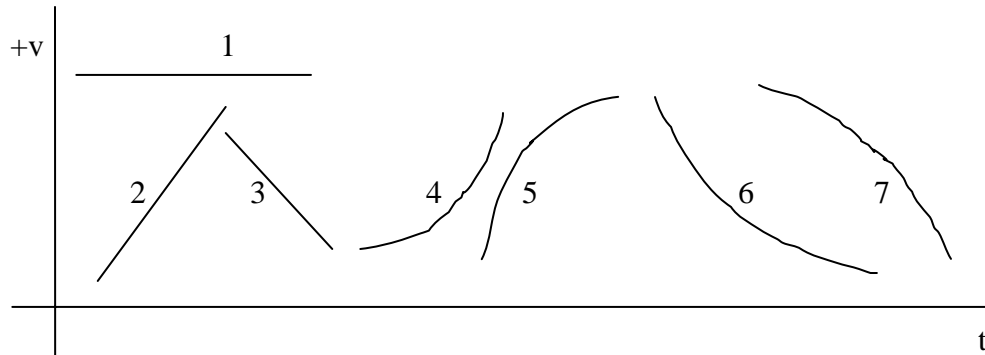
$$\begin{aligned} \text{slope} &= | \text{acceleration} | \\ \text{area under the curve} &= \text{distance covered} \end{aligned}$$

Velocity vs. Time Graph (for 1-D motion)



$$\begin{aligned} \text{slope} &= \text{acceleration} \\ \text{area under the curve} &= \text{displacement} \end{aligned}$$

Velocity vs. Time Graph (for 1-D motion)



1. slope zero \rightarrow constant velocity
2. constant positive slope (increasing velocity) \rightarrow constant accelerated motion
3. constant negative slope (decreasing velocity) \rightarrow constant negative accelerated motion
4. increasing positive slope (increasing velocity) \rightarrow non uniform accelerated motion (increasing acceleration)
5. decreasing positive slope (increasing velocity) \rightarrow non uniform accelerated motion (decreasing acceleration)
6. decreasing negative slope (decreasing velocity) \rightarrow non uniform negative accelerated motion (decreasing negative acceleration)
7. increasing negative slope (decreasing velocity) \rightarrow non uniform negative acceleration (increasing negative acceleration)

kna02vd1

knm02vd1

kna03vd1

knm03vd1

kna03vd2

knm03vd2

kna04vd1

knm04vd1

knm06an1

knm06an2

knm05vd1

2.7 Some Types of Motion

- Rectilinear motion with constant acceleration
- Projectile motion
- Circular motion
- Relative motion

2.7.1 Rectilinear motion with constant acceleration

Direction of *acceleration* is *parallel* to the *path*

Motion along X-axis

$$(\bar{v}_{av})_x = \frac{\Delta x}{\Delta t} \quad (\text{general})$$

The following equations are *valid* only if *a_x is constant*.

$v_x = v_{0x} + a_x \cdot t \quad \Rightarrow \quad v_x = v_{0x} + a_x \cdot (t - t_0)$
$v_x^2 = v_{0x}^2 + 2 \cdot a_x \cdot x \quad \Rightarrow \quad v_x^2 = v_{0x}^2 + 2 \cdot a_x \cdot (x - x_0)$
$x = v_{0x} \cdot t + \frac{1}{2} a_x \cdot t^2 \quad \Rightarrow \quad (x - x_0) = v_{0x} \cdot (t - t_0) + \frac{1}{2} a_x \cdot (t - t_0)^2$
$(\bar{v}_{av})_x = \frac{v_{0x} + v_0}{2}$

(Note : Proper signs have to be used for all a, v and x)

Example 2.5

Using calculus, find (a) the velocity and acceleration at any time and (b) the position, velocity and acceleration at $t = 2$ s for a particle moving along the x-axis with its x coordinate (in meter) being given by $x = 2t^3 - 10t^2 + 45$. (c) Is it speeding up or slowing down at that instant?

solution

$x = 2t^3 - 10t^2 + 45$ m is the position of the given particle at any time. As the motion is one dimensional, we need to consider only the signed scalars of the motional characteristics

- (a) We may determine the velocity and the acceleration at any time in such a way that

$$v = \frac{dx}{dt} = \frac{d}{dt}(2t^3 + 10t^2 + 45) = 6t^2 + 20t \quad \text{m s}^{-1}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(6t^2 + 20t) = 12t + 20 \quad \text{m s}^{-2}$$

- (b) At $t = 2$ s, using the respective expressions, we have

$$\text{the position} \quad x = 2t^3 - 10t^2 + 45 = 2(2)^2 + 45 = 21 \text{ m}$$

$$\text{the velocity} \quad v = 6t^2 - 20t = 6(2)^2 - 20(2) = -16 \text{ m s}^{-1}$$

$$\text{the acceleration} \quad a = 12t - 10 = 12(2) - 20 = 4 \text{ m s}^{-2}$$

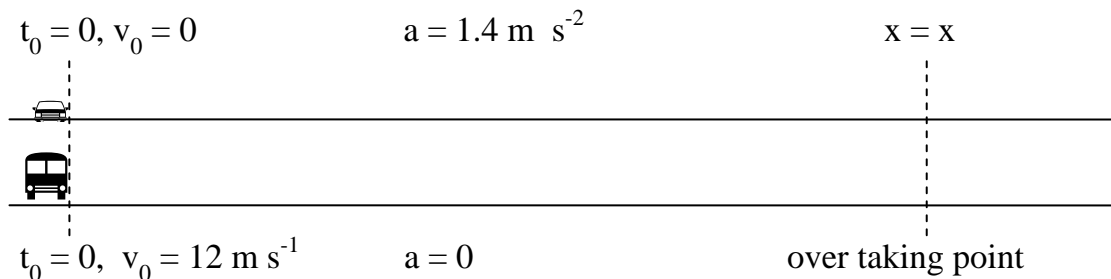
- (c) It is slowing down at that instant as the direction of the acceleration is opposite to that of motion.

Example 2.6

Just as a car starts to accelerate from rest with acceleration 1.4 m s^{-2} , a bus moving with constant speed of 12 m s^{-1} passes it in parallel lane. (a) How long before the car overtakes the bus? (b) How fast will the car then be going? (c) How far will the car then be gone?

Solution :

First of all, the x-axis was chosen to lie along the parallel lanes and the origin was taken to be the point at which the car started. As the motion is one-dimensional we need to consider only the signed scalars of the motional characteristics. In this problem, the car moved with constant acceleration whereas the bus did with constant speed.



For the case of the bus, we have $x - x_0 = v_0(t - t_0) + \frac{1}{2} a (t - t_0)^2$

$$x - 0 = 0(t - 0) + \frac{1}{2} (1.4) (t - 0)^2$$

which gives $x = 0.7 t^2$ -----(1)

For the case of the bus, we have $x - x_0 = v_0 (t - t_0)$

$$x - 0 = 12 (t - 0), \text{ which gives } x = 12 t \text{ -----(2)}$$

Since both of them have the same starting point, the same end point and the same origin, the two equations (1) and (2), $x = 0.7 t^2 = 12t$. Thus, $t = 17.14$ s. Therefore, the elapsed time before the car overtakes the bus is 17.14 s.

(b) To find the speed of car while overtaking, we have
 $v = v_0 + a (t - t_0) = 0 + 1.4 (17.14 - 0) = 24 \text{ m s}^{-1}$.

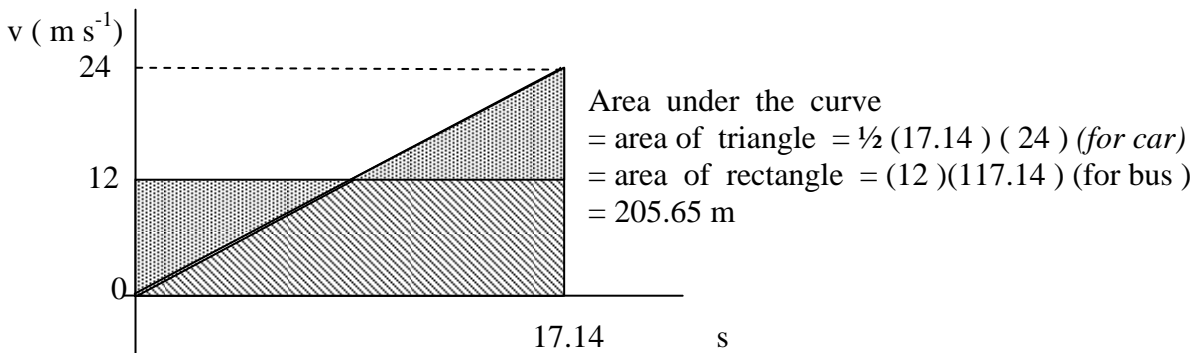
Thus, the car then will be going with the speed of 24 m s^{-1} .

(c) To find the distance moved of the car, we have

$$x = 12 t = 12(17.14) = 205.65 \text{ m.}$$

Therefore, the car then will be gone 205.65 m from its initial point.

Graphical method



2.7.2 Freely Falling Body

In the absence of air resistance, a body falling under the influence of the earth's gravitational attraction near the earth-surface gained (nearly) constant acceleration, regardless of its size, weight and composition.

Free-fall motion - Motion of a body under only the influence of earth's gravitational field (force).

The kinematics equations for straight line motion with constant acceleration can be used for free-fall motion. In such a case acceleration due to gravity “g” [$g = 9.81 \text{ ms}^{-2} \sim 10 \text{ ms}^{-2}$] has to be used instead of acceleration.

Note. A proper sign convention according to the selected reference frame must be used. Because of $g = 10 \text{ ms}^{-2}$, magnitude of velocity of freely falling body will change, may be increased or decreased, 10 ms^{-1} in every each second.

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Example 2.7 A body released from rest falls freely. Compute its position and velocity after 1, 2, 3 and 4 s. Take the origin O at the elevation of the starting point, the y-axis vertical, and the upward direction as positive.

Solution

Since the problem is essentially one dimensional we can use the signed scalars of motional characteristics. The initial coordinate y_0 and the initial velocity v_0 are both zero. The acceleration is downward in y direction, so that

$$a = -g = -32 \text{ ft s}^{-2}$$

To find the position at any instant, we have

$$y - y_0 = v_0 (t - t_0) + \frac{1}{2} a (t - t_0)^2$$

$$y - 0 = 0 (t - 0) + \frac{1}{2} (-32) (t - 0)^2$$

$$\text{which gives } y = -16 t^2 \text{ -----(1)}$$

and the velocity at any instant can be found from

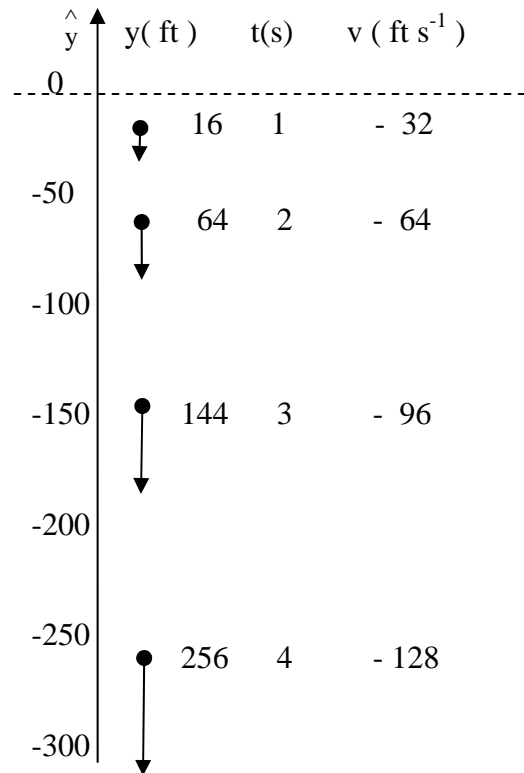
$$v = v_0 + a (t - t_0)$$

$$v = 0 + (-32) (t - 0) = -32 t \text{ ----- (2)}$$

When $t = 1 \text{ s}$, $y = 16 \text{ ft}$ and $v = -32 \text{ ft s}^{-1}$

The body is therefore 16 ft below the origin (y is negative) and has a downward velocity (v being negative) of 32 ft s^{-1} .

The position and velocities for 2, 3 and 4 s are found in the same way and are shown in the associated figure.



Example 2.8 A ball is thrown (nearly) vertically upward from the cornice of a building, leaving the thrower's hand with a speed of 14 m s^{-1} and just missing the cornice on the way down. Find (a) the position and the velocity of the ball, 1 s and 4 s after leaving the thrower's hand ; (b) the velocity when the ball is 6 m above its starting point; (c) maximum height reached and the time at which it is reached.

Solution :

Taking the origin at the height at which the ball leaves the thrower's hand, the vertical is taken along the y-axis and the upward direction as positive. Remember that only the signed scalars of the motional characteristics are needed to be considered. The initial coordinate $y_0 = 0$ at $t_0 = 0$ and the acceleration is downward in y direction so that

$$a = -g = -10 \text{ m s}^{-2}$$

Since the initial velocity $v_0 = 14 \text{ m s}^{-1}$, the velocity at any time is

$$\begin{aligned} v &= v_0 + a(t - t_0) \\ &= 14 + (-10)(t - 0) \\ &= 14 - 10t \end{aligned} \quad \text{----- (1)}$$

The position coordinate at any time is

$$\begin{aligned} y - y_0 &= v_0(t - t_0) + \frac{1}{2} a(t - t_0)^2 \\ y &= 0 + 14(t - 0) + \frac{1}{2}(-10)(t - 0)^2 \\ y &= 14t - 5t^2 \end{aligned} \quad \text{----- (2)}$$

(a) When $t = 1 \text{ s}$, the position of the ball is

$$y = 14t - 5t^2 = 14 - 5 = +9 \text{ m}$$

and its velocity is

$$v = 14 - 10t = 14 - 10(1) = +4 \text{ m s}^{-1}$$

The ball therefore is 9 m above the origin (y is positive) and its velocity is 4 m s^{-1} upward (v being positive).

$$\begin{aligned} \text{When } t = 4 \text{ s, } \quad y &= 14t - 5t^2 = 14(4) - 5(4)^2 = -24 \text{ m} \\ \quad \quad \quad v &= 14 - 10t = 14 - 10(4) = -26 \text{ m s}^{-1}. \end{aligned}$$

The ball, therefore, is 24 m below the origin (y is negative) and its velocity is 26 m s^{-1} downward (v being negative).

(b) The ball is 6 m above its starting point so that $y = +6 \text{ m}$

The velocity at that position is obtained from

$$v^2 = v_0^2 + 2 a (y - y_0)$$

$$= (14)^2 + 2 (-10) (+6 - 0)$$

and we get $v = \pm 8.7 \text{ m s}^{-1}$

The ball therefore passes that point twice (v is positive or negative); once on the way up and once on the way down. The velocity on the way up is $+8.7 \text{ m s}^{-1}$ and on the way down it is -8.7 m s^{-1} .

(c) At the highest point $v = 0$, Hence the maximum height reached is obtained from

$$v^2 = v_0^2 + 2 a (y - y_0)$$

$$0 = (14)^2 + 2 (-10) (y - 0)$$

and we get $y = 9.8 \text{ m}$

(d) The time at which the ball reached can be found from

$$v = 14 - 10 t$$

$$0 = 14 - 10 t$$

which gives $t = 1.4 \text{ s}$

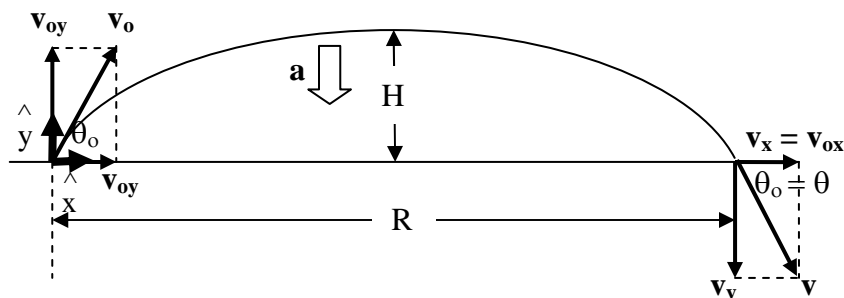
2.7.3 Motion of a Plane

It is a two dimensional motion.

2.7.3.1 Motion of a Projectile

Projectile motion is a *combination of two independent motion*: (*horizontal*) motion with *constant velocity* and (*vertical*) motion with *constant acceleration*. The path of the projectile is a *parabola*.

note A projectile motion will occur whenever *direction of motion* and *direction of acceleration* of moving object form any angle except 0° or 180° . In such motion, acceleration can be caused by any kind of net force. Moreover the projectile will *move in the plane formed by velocity vector and acceleration vector*.



$$\vec{a} = -\hat{y} g \quad ; \quad \vec{a} = \vec{a}_x + \vec{a}_y = \hat{x} 0 + \hat{y} (-g)$$

$$\text{at } t=0, \vec{v} = v_0, \quad \vec{v} = \hat{x} v_{0x} + \hat{y} v_{0y} = \hat{x} v_0 \cos \theta_0 + \hat{y} v_0 \sin \theta_0$$

Motion in x - direction

$$a_x = 0$$

$$v_x = v_{0x} = v_0 \cos \theta_0$$

$$x = v_{0x} \cdot t = v_x t$$

Motion in y - direction

$$a_y = -g,$$

$$v_{0y} = v_0 \sin \theta_0$$

$$v_y = v_{0y} + a_y t$$

$$v_y^2 = v_{0y}^2 + 2 \cdot a_y \cdot y$$

$$y = v_{0y} \cdot t + \frac{1}{2} a_y \cdot t^2$$

Note Time interval for any motion viz vertical, horizontal and along the path are ever the same.

Time required to reach the maximum height, 't'

$$v_y = 0$$

$$0 = v_0 \sin \theta_0 + (-g) t$$

$$v_y = v_{0y} + a_y t$$

$$t = \frac{v_0 \sin \theta_0}{g}$$

Time of flight 'T' [to return same altitude]

$$T = 2t = \frac{2 v_0 \sin \theta_0}{g}$$

(since time of rise is equal to time of fall.)

(or) $v_y = v_{0y} + a_y t$

$$-v_0 \sin \theta_0 = +v_0 \sin \theta_0 + (-g) \cdot T$$

$$T = \frac{2 v_0 \sin \theta_0}{g}$$

(only true for the case of same altitude)

Maximum height 'H'

at the maximum height $v_y = 0$

$$v_y^2 = v_{0y}^2 + 2 a_y \cdot y$$

$$0 = (v_0 \sin \theta_0)^2 + 2(-g)(H)$$

$$H = \frac{v_0^2 \sin^2 \theta_0}{2g}$$

Range (horizontal distance covered for same altitude) 'R'

$$x = v_{0x} \cdot t = v_x t$$

$$R = (v_0 \cos \theta_0)(T) = (v_0 \cos \theta_0) \left(\frac{2v_0 \sin \theta_0}{g} \right) = \frac{v_0^2 2 \sin \theta_0 \cos \theta_0}{g}$$

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

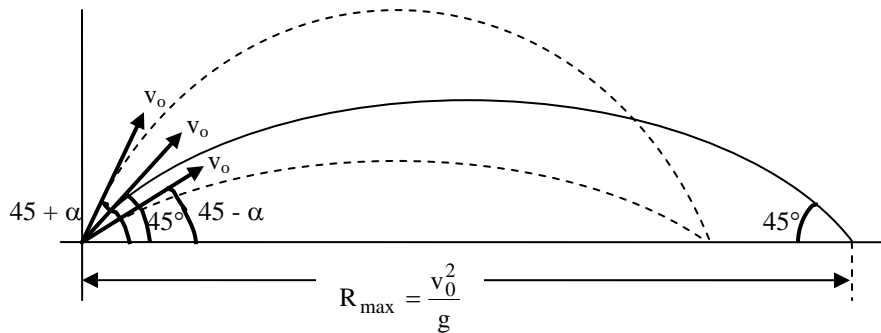
[since, $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$]

To reach maximum range, $\sin 2\theta_0$ must be maximum.

$$\text{i.e. } \sin 2\theta_0 = 1 = \sin 90^\circ$$

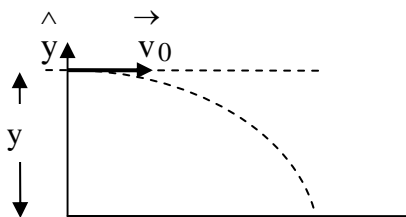
$$\theta_0 = 45^\circ$$

$$R_{\max} = \frac{v_0^2}{g}$$



Path of a projectile motion

(a) **Launch horizontally**



$$y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$y = 0 + \frac{1}{2} (-g) t^2$$

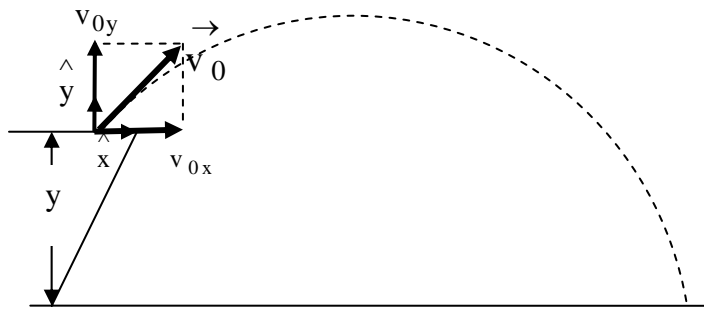
$$y = -\frac{1}{2} g t^2$$

$$x = v_0 t, \quad t^2 = \frac{x^2}{v_0^2}$$

$$\begin{aligned}\vec{v}_0 &= \hat{x} v_{0x} + \hat{y} v_{0y} \\ &= \hat{x} v_0 \cos \theta_0 + \hat{y} v_0 \sin \theta_0\end{aligned}$$

$$y = -\frac{g}{2v_0^2} \cdot x^2$$

(b) **Launch with an angle with respect to horizontal**



$$y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$y = v_0 \sin \theta_0 t + \frac{1}{2} (-g) t^2$$

$$x = v_{0x} \cdot t = v_x t = v_0 \cos \theta_0 \cdot t$$

$$t = \frac{x}{v_0 \cos \theta_0}$$

$$t^2 = \frac{x^2}{v_0^2 \cos^2 \theta_0}$$

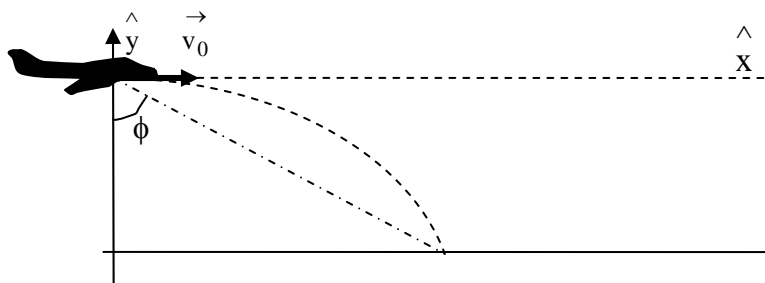
$$y = v_0 \cdot \sin \theta_0 \cdot \frac{x}{v_0 \cos \theta_0} - \frac{g}{2} \cdot \frac{x^2}{v_0^2 \cos^2 \theta_0}$$

$$y = \tan \theta_0 \cdot x - \frac{g}{2 v_0^2 \cos^2 \theta_0} \cdot x^2$$

Both equations for case (a) and case (b) indicate the trajectory of a projectile is **parabolic**.

Example 2.9

A bomber is flying at a constant velocity of 370 m s^{-1} at an elevation of $15,850 \text{ m}$ toward a point directly above its target. At what angle of sight (shown in figure) should a bomb be released to strike the target?



Solution :

We choose a reference frame fixed with respect to the earth, its origin '0' being the bomb release - point. The motion of the bomb at the instant release is the same as the bomber.

Hence the initial velocity of bomb v_0 (at $t_0 = 0$) is horizontal and its magnitude is 370 m s^{-1} .

The angle of projection $\theta_0 = 0$

$$\text{Given at } t_0 = 0, \quad x_0 = 0, \quad y_0 = 0$$

$$v_{0x} = 370 \text{ m s}^{-1}, \quad v_{0y} = 0$$

Its acceleration components are $a_x = 0$ and $a_y = -g$

The time of fall (t) for $y = 15850 \text{ m}$ is obtained from equation

$$y = v_{0y} t + \frac{1}{2} (-g) t^2$$

$$-15850 = 0 - \frac{1}{2} (10) t^2$$

$$t = 56.3 \text{ s}$$

The horizontal distance travelled by the bomb in this time is given by

$$x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$= 370 \times 56.3$$

$$= 20831 \text{ m.}$$

From the associated figure, the angle of sight is then

$$\phi = \tan^{-1} \left(\frac{x}{y} \right) = \tan^{-1} \left(\frac{20831}{15850} \right) = 52.7^\circ$$

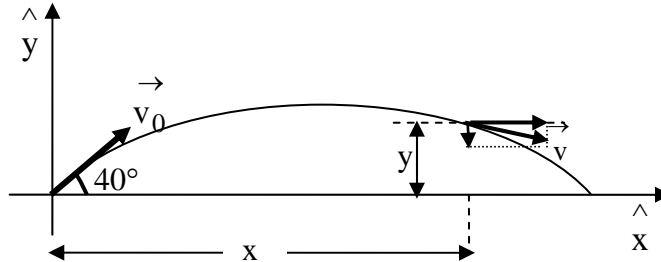
note The time of fall of the bomb does not depend on the speed of the plane for a horizontal projection.

Example 2.10

A gun fires a bullet with a muzzle velocity of 200 m s^{-1} at an angle of 40° with the ground. Find the velocity and position of the bullet after 20 s. Also find the range and time required for the bullet to return to ground.

Solution

We choose the xy -reference frame with the x -axis horizontal and y -axis vertical its origin will be taken as the point from which bullet begins its free flight.



The initial conditions of motion are $v_0 = 200 \text{ m s}^{-1}$ and $\theta_0 = 40^\circ$

Hence, at $t_0 = 0$, $x_0 = 0$ and $y_0 = 0$

$$v_{0x} = v_0 \cos \theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin \theta_0$$

The components of acceleration are $a_x = 0$ and $a_y = -g$.

The components of velocity at any later time 't' can be obtained from equation

$$v = v_0 + a t \quad \text{as}$$

$$v_x = v_{0x} \quad \text{and} \quad v_y = v_{0y} - g t = 128.6 - 10 t \quad \text{m s}^{-1}$$

So that at $t = 20 \text{ s}$, they are

$$v_x = 153.2 \text{ m s}^{-1} \quad \text{and} \quad v_y = -71.4 \text{ m s}^{-1}$$

A negative sign means that the bullet is descending at that instant with a speed of bullet

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(153.2)^2 + (-71.4)^2} = 169.02 \text{ m s}^{-1}$$

The directions of velocity at $t = 20 \text{ s}$ can be expressed an angle of

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{-71.4}{153.2} \right) = -24.98^\circ \text{ with the positive } x\text{-axis.}$$

The position of the point P (at $t = 20 \text{ s}$) is obtained by using equation

$$x = v_{0x} t = 153.2 \times 20 = 3064 \text{ m} \quad \text{and}$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$= (128.6 \times 20) - \frac{1}{2} \times 10 (20)^2 = 572 \text{ m.}$$

The time required for the bullet to return to ground is obtained by using $y = 0$ and equation

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$0 = v_{0y} T - \frac{1}{2} g T^2$$

$$T = \frac{2 v_{0y}}{g} = \frac{2 \times 128.6}{10} = 25.72 \text{ s.}$$

The range of bullet R (the total horizontal distance covered during the time of flight) is obtained by substituting 'T' in equation $x = v_{0x} t$

$$R = v_{0x} T = 153.2 \times 25.72 = 3940.3 \text{ m.}$$

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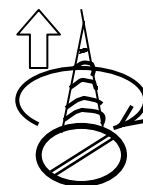
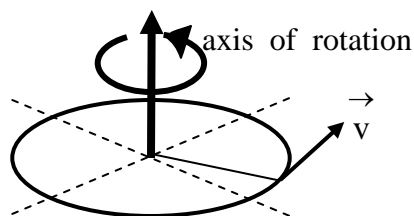
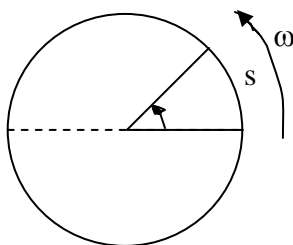
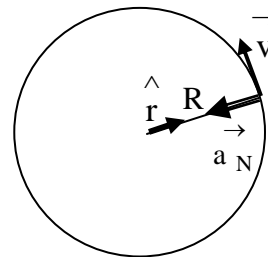
2.6.3-2 Circular Motion

In uniform circular motion a particle moves in a circular path with constant speed (magnitude of velocity) but continuously changing direction of velocity. Acceleration of the particle at each point is directed toward the centre of the circle (i.e. centripetal acceleration). Its magnitude depends on the speed and the radius of the circle.

For uniform circular motion

$$\vec{a}_N = \vec{a}_C = \vec{a}_R = (-\hat{r}) \frac{v^2}{R} = (-\hat{r}) R \omega^2$$

\hat{r} = *unit vector* that points radially outward.



Relation between linear and (rotational) angular parameters

parameter	linear	SI unit	rotational	SI unit	relation
displacement	s	m	θ	rad	$s = r \theta$
average velocity	v_{av}	$m s^{-1}$	$\omega_{av} = \frac{\Delta \theta}{\Delta t}$	$rad s^{-1}$	
instantaneous velocity	v	$m s^{-1}$	$\omega = \frac{d\theta}{dt}$	$rad s^{-1}$	$v = r \omega$
average acceleration	$a_{av} = a_{T av}$	$m s^{-2}$	$\alpha_{av} = \frac{\Delta \omega}{\Delta t}$	$rad s^{-2}$	
instantaneous acceleration	$a = a_T$	$m s^{-2}$	$\alpha = \frac{d\omega}{dt}$	$rad s^{-2}$	$a_T = r \alpha$

Comparison of linear and (rotational) angular motion with constant acceleration**Straight-line motion with constant linear acceleration**

$$a = \text{constant}$$

$$v = v_0 + a t$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2} (v_0 + v) t$$

$$\omega = \omega_{av} = \frac{2\pi}{T} \quad (\text{If } \omega \text{ is constant})$$

$$v = v_{av} = \frac{2\pi r}{T} \quad (\text{If speed is constant})$$

Fixed axis rotation with constant angular acceleration

$$\alpha = \text{constant}$$

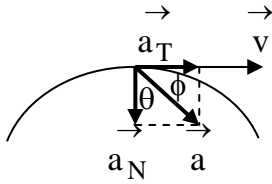
$$\omega = \omega_0 + \alpha t$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

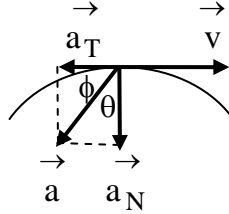
$$\theta - \theta_0 = \frac{1}{2} (\omega_0 + \omega) t$$

In general



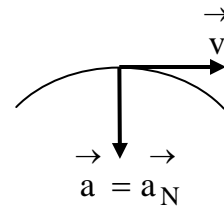
speed is increasing

$$a_N = \frac{v^2}{r} = r\omega^2;$$



speed is decreasing

$$a_T = \frac{v - v_0}{t} = r\alpha$$



speed is constant

$$a = a_N$$

$$a = \sqrt{a_N^2 + a_T^2} = \sqrt{(r\omega^2)^2 + (r\alpha)^2}$$

$$\phi = \tan^{-1} \left(\frac{a_N}{a_T} \right) = \tan^{-1} \left(\frac{\omega^2}{\alpha} \right)$$

$$\theta = \tan^{-1} \left(\frac{a_T}{a_N} \right) = \tan^{-1} \left(\frac{\alpha}{\omega^2} \right)$$

Example 2.11 An earth - satellite revolves in a circular orbit at a height of 300 km above the earth's surface where $g = 8.94 \text{ m s}^{-1}$. Assuming that the earth's radius to be 6,400 km, find (a) the speed of the satellite, and (b) Its period of revolution T. (c) If it were a communication satellite in a synchronous orbit of 35,800 km altitude, orbiting the earth at the same rate that the earth turns so that it would be 'fixed' with respect to the observer on the earth, find 'g' at this altitude.

Solution

Like any free object near the earth's surface the satellite has an acceleration 'g' towards the centre of the earth. This acceleration causes to hold it in the circular orbit so that the centripetal acceleration is taken to be $a_R = g = 8.94 \text{ m s}^{-2}$.

(a) The speed of the satellite can be found by using

$$a_R = \frac{v^2}{r}$$

$$g = \frac{v^2}{R+h} \quad \Rightarrow \quad 8.94 = \frac{v^2}{(6700 \times 10^3)}$$

which gives speed of the satellite $v = 7.74 \text{ km s}^{-1}$

This speed can be regarded as a constant since the satellite revolves in the orbit of fixed radius and 'g' is constant. Hence its motion is uniform circular motion.

(b) To find its period of revolution, we have

$$\begin{aligned} T &= \frac{2\pi r}{v} = \frac{2\pi(6700)}{7.74} \\ &= 6.436 \times 10^3 \text{ s} \\ &= 90.6 \text{ min} \end{aligned}$$

(c) Since the satellite orbits at the same rate that the earth turns so that $T = 24 \text{ h}$, its centripetal acceleration can be found by using the relations

$$\begin{aligned} a_R &= r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 \\ &= \frac{4\pi^2(R+h)}{T^2} = \frac{4\pi^2(6400+35800) \times 10^3}{(24 \times 60 \times 60)^2} \\ &= 0.224 \text{ m s}^{-2} \end{aligned}$$

This acceleration is given by the earth so that it is the acceleration of gravity at such an altitude. It is very much less than the acceleration of gravity near the earth surface of 9.8 m s^{-2} , by the factor of 2.28×10^{-2} .

2.6.4 Motion in Different Frames (Relative motion)

The figure illustrates, a body 'p' moves relative to a body (or reference frame) B, and B moves relative to the body (or reference frame) A.

V_{BA}	=	Velocity of frame B with respect to frame A
r_{BA}	=	Position of frame B with respect to frame A
r_{PA}	=	Position of P with respect to frame A
r_{PB}	=	Position of P with respect to frame B

According to diagram,

$$r_{PA} = r_{PB} + r_{BA}$$

In general,

- = velocity of A with respect to D
- = velocity of A with respect of B
- = velocity of B with respect of C
- = velocity of C with respect of D

Note ;

Strateg for description of relative velocity with its components

1. Write each velocity with a double subscript in the proper order, meaning ‘velocity of (first subscript) relative to (second subscript)’
2. When adding relative velocities, the first letter of any subscript is to be the same as the last letter of the preceding subscript.
3. The first letter of the subscript of the first velocity in the sum, and the second letter of the subscript of the last velocity, are the subscripts, in that order of the relative velocity represented by the sum.

Example 2.13 Observer ‘O’ drops a stone from the top of a skyscraper. Observer ‘O’ riding in an elevator, starts down from the top of the skyscraper at the instant when the stone is dropped. The elevator accelerates very quickly to a downward velocity of magnitude $v = 5.0 \text{ m s}^{-1}$ and then maintains that velocity steadily. At the time $t = 3.0 \text{ s}$ after the stone is dropped, find the position, the velocity and the acceleration of the stone relative to ‘O’. Then find the position, the velocity and the acceleration of the stone relative to O’.

Solution :

Observer O is located at the origin of the unprimed frame and observer O’ at that of the primed frame. Since the problem is essentially one-dimensional, we can use only the signed scalars representations of vectors in the y-axis.

Since the observer 'O' is stationary relative to the earth, the acceleration of the freely falling stone seen by O is that of gravity.

Taking the upward direction as positive y-direction, we have

$$a_y = -g = -9.8 \text{ m s}^{-2}$$

As seen by the observer O, $t_0 = 0$, $y_0 = 0$ and $v_{0y} = 0$ so that the position of the stone at the end of 3 s is obtainable from the equation

$$y - y_0 = v_{0y} (t - t_0) + \frac{1}{2} a_y (t - t_0)^2$$

$$y - 0 = 0 + \frac{1}{2} (-9.8) (3 - 0)^2$$

which gives $y = -44.1 \text{ m}$

Thus, the stone is located 44.2 m below the observer O, at the end of 3 s. And then we can find its velocity relative to the observer O, using the equation

$$\begin{aligned} v_y &= v_{0y} + a_y (t - t_0) \\ &= 0 + (-9.8) (3 - 0) \\ &= -29.4 \text{ m s}^{-1} \end{aligned}$$

Here the negative sign means its velocity at the end of 3 s is downward. Since O' moves at constant velocity with respect to O, the position of the stone seen by the observer O' can be calculated from the Gallilean position transformation

$$\begin{aligned} y' &= y - V_y t \\ &= (-44.1) - (-5)(3) = -29.1 \text{ m} \end{aligned}$$

The stone is therefore located 29.1 m below the observer O' at the end of 3 s. As the Gallilean velocity transformation gives use the velocity with respect to O', we have

$$\begin{aligned} v_y' &= v_y - V_y \\ &= (-29.4) - (-5) = -24.4 \text{ m s}^{-1} \end{aligned}$$

According to the Gallilean acceleration transformation $a_y' = a_y$ This

means the observer O' sees the stone to have the same acceleration as that seen by the observer O.

Thus, we have $a_y' = -g = -9.8 \text{ m s}^{-2}$

Example 2.14 In order to reach his destination on schedule an airline pilot whshes to fly over the ground at a 'ground speed' of 250 m s^{-1} along a north-

easterly direction (in other words, 45° north of east). The weather bureau tells him that the wind is blowing due eastward with a speed of 30 m s^{-1} . Find the required 'air speed' of the airplane (its speed relative to the air through which it flies, read by the pilot on his airspeed indicator) and the required 'heading' (the geographical direction in which the nose of the plane should be oriented read by the pilot on his compass), if the plane is to reach the right place at the right time.

Solution

Although it is possible to think of the pilot as both the moving observer and the observed moving body, you may find it easier to analyse the problem if you make a separation as follows. Imagine an observer O' in a balloon which floats along with the wind. His velocity with respect to the observer O on the ground is $V = v_{AG} = 30 \text{ m s}^{-1}$ (eastward). Since the airplane is also carried along by the wind as it moves through the air, O' will observe the proper air speed and heading which comprise the velocity $v' = v_{PA}$.

At the same time O observes the desired ground-speed and the course, $v = V_{PG} = 250 \text{ m s}^{-1}$ (northeastward)

Since the problem is essentially two-dimensional, you need only x and y components

$$v'_x = v_x - V_x \quad \text{and} \quad v'_y = v_y - V_y \quad \text{----- (1)}$$

To save some work, take the positive x -direction to be eastward, that is, the direction in which O sees O' to be moving. The components of V then become

$$V_x = V = 30 \text{ m s}^{-1} \quad \text{and} \quad V_y = 0$$

Now calculate the x and y components of the desired ground velocity v . You have

$$v_x = v \cos 45^\circ = 250 \cos 45^\circ = 177 \text{ m s}^{-1}$$

$$\text{and } v_y = v \sin 45^\circ = 250 \sin 45^\circ = 177 \text{ m s}^{-1}$$

Inserting the numerical values of v_x and V_x in eq (1), you obtain

$$v'_x = 177 - 30 = 147 \text{ m s}^{-1}$$

Similarly, substituting the numerical values of v_y and V_y you get

$$v'_y = 177 - 0 = 177 \text{ m s}^{-1}$$

You can now calculate the magnitude and direction of the velocity v' of the plane as seen by O' in the ballon (and also of the plane with respect to the plane with respect to the air as measured by the pilot's instruments). You have for the magnitude

$$\text{m s}^{-1}$$

For the direction you have

Since the positive x direction has been taken to be eastward, this angle signifies a direction 50.3° north of east, and the pilot should fly the plane with a velocity.

$$= 230 \text{ m s}^{-1} (50.3^\circ \text{ north of east }) \text{ through the air.}$$

CHAPTER III

3. Dynamic of a Particle

Dynamic : The relationship of motion to the forces that **cause** it.(i.e. the study of **cause** of accelerated motion.)

3.1 Newton's Law of Motion

Newton's First Law

In the absence of a **resultant (or net) force**, a body at rest will remain at rest, and a body in motion continue moving in a **straight line at constant speed**.

In symbol : If $\vec{F}_{\text{net}} = 0$, then $\Delta \vec{v} = 0$. (or $\vec{v} = \text{constant}$ or $\vec{a} = 0$)

note Newton's first law is often called the **Law of Inertia**.

- **Inertia** is the *reluctance* of a body to change its state of rest or motion.
- Inertia is quantified by mass, i.e., the larger the mass, the greater the inertia.

Newton's Second Law

When a **resultant force acts on particle** (i.e. constant mass), it will be accelerated in the direction of the resultant force with an acceleration that is proportional to the magnitude of the force.

In symbol $\vec{a} \propto \vec{F}_{\text{net}}$
 $\vec{F}_{\text{net}} = k m \vec{a}$

$k = 1$, if 1 kg mass gets an acceleration 1 m s^{-2} owing to 1N force.

$\therefore \vec{F}_{\text{net}} = m \vec{a}$ (for constant mass)

In general -

The rate of change of momentum is directly proportional to the net force acting on it and takes place in the direction of the force.

$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$ (where $\vec{p} = m\vec{v} = \text{momentum}$)

$\vec{F}_{\text{net}} = m \frac{d\vec{v}}{dt} + \vec{v} \frac{dm}{dt}$

If mass of the body is constant , $\frac{dm}{dt} = 0$

$$\vec{F}_{\text{net}} = m \frac{d \vec{v}}{d t} = m \vec{a} .$$

note Newton's first law is the *special case* of Newton's second law for the case of *zero net force*.

i.e. $\vec{F}_{\text{net}} = m \vec{a}$ (second law)

if $\vec{F}_{\text{net}} = 0$, $m \vec{a} = 0$

but $m \neq 0$ $\therefore \vec{a} = 0$ (first law)

or $\vec{v} = \text{constant}$ or $\Delta \vec{v} = 0$.

Newton's Third Law

Action force and reaction force have equal magnitude and opposite direction.

In symbol $\vec{F}_{AB} = - \vec{F}_{BA}$

where \vec{F}_{AB} = force exerted by A on B.

\vec{F}_{BA} = force exerted by B on A.

note Action and Reaction cannot cancel to each other since these forces exerted on different objects.

- Even though $\vec{F}_{AB} = - \vec{F}_{BA}$ is correct, $\vec{F}_{AB} + \vec{F}_{BA} = 0$ is incorrect.
- Newton's law of motion are valid *only* for *inertial frame of reference*.
- A frame of reference in which Newton's first law is satisfied, is called ***an inertial frame of reference*** (or) Acceleration of inertial frame of reference is zero.

Inertia Inertia is the **reluctance** of a body to change its state of rest or motion.

Mass Mass is a quantitative measure of inertia. (It is also known as *inertial mass*). It is a scalar quantity. Its SI unit is 'kg'. Mass is independent of the place where it is kept.

Force Force is an **influence** which can cause the change in state of motion of the body if a force is acting on that body. It is a

vector quantity. Its SI unit is **newton 'N'**. Magnitude of a force (net force) can be found by Newton's second law. $F = m a$. **1 N force** is defined as the force needed to produce an acceleration of 1 m s^{-2} for mass of 1 kg. (i.e. $1\text{N} = 1 \text{ kg} \times 1 \text{ m s}^{-2}$)

note Newton's first law of motion gives definition of force. It states if there is no (net) force there will be no acceleration (or) the state of motion, may be except position, will not be changed.

Newton's second law of motion gives magnitude and direction of force (net force). Unit of force can also be defined.

Newton's third law of motion gives nature of forces. To achieve the force two or more bodies are required. i.e. No single body system cannot create a force.

3.2 Types of Forces

The followings are some common types of forces.

(a) Gravitational Forces

Any two masses are attracted to each other by gravitational force. *Newton's Gravitational Law*

The **attractive force** between two masses is directly proportional to the product of masses and inversely proportional to the distance between them.

$$\text{In symbol } F \propto \frac{m_1 m_2}{r^2} \Rightarrow F = G \frac{m_1 m_2}{r^2}$$

where 'G' is known as **universal gravitational constant** and has a value of $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

note The attracting force by the earth on a body is known as **weight** of that body.

$$\text{i.e. } w = F = G \frac{M_E m}{R^2} = m g$$

where

M_E	=	mass of the earth
m	=	mass of the body
R	=	average radius of the earth
g	=	acceleration due to gravity

Weight depends on the **location** where the object is placed.

(b) Electrical Forces

These are forces between **electric charges**. These forces can be **attractive** or **repulsive**. Electrical forces between two point charges obey **Coulomb's Law**.

Like charges repel and unlike charges attract to each other.

(c) Magnetic Forces

These are forces that act on **magnetic materials** and on **electric current carrying conductors** or on **moving electric charges** whose directions of motion are not parallel or anti-parallel to the magnetic field.

Like poles repel, unlike poles attract to each other.

(d) Frictional Forces (friction)

These are forces which **resist motion** or **tendency of motion**. Frictional forces are *dissipative* in nature. i.e., these forces can cause the lost of energy. Friction force is a **contact** force and **always tangential to the surface**. Its direction is always **opposite** to the **direction motion** or **tendency of motion**.

Force of friction \Rightarrow - static friction

\Rightarrow - kinetic friction \rightarrow sliding friction

\rightarrow rolling friction

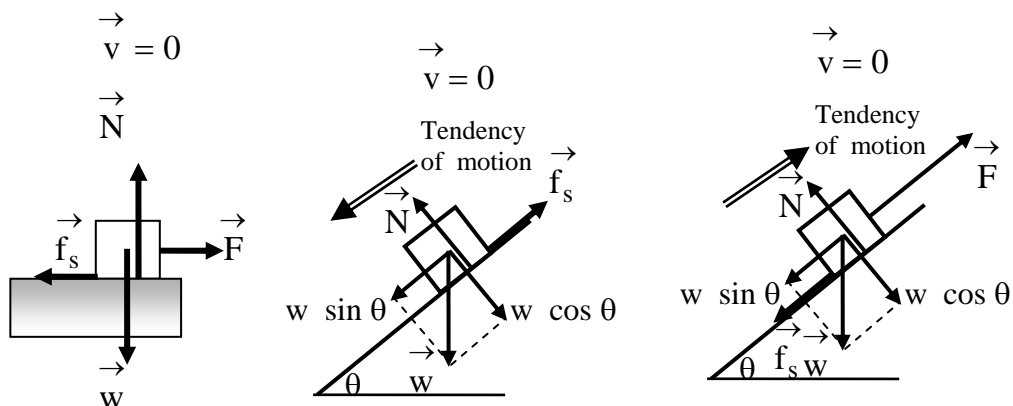
Static friction (f_s) can vary from zero to limiting force of static friction ($f_{s \max}$). [i.e. $0 < f_s < f_{s \max}$]. Limiting static friction (limiting force of static friction is found to be dependent on the **nature** and **kind of pair of surfaces** and the **normal force** to the contact surface.

i.e. $f_s = \mu_s N$

where $f_s =$ limiting force of static friction $= f_{s \max}$

$\mu_s =$ coefficient of static friction

$N =$ normal reaction



note Even though $f_s = \mu_s N$ is correct $\vec{f} = \mu_s \vec{N}$ is incorrect since \vec{f} and \vec{N} have different directions.

Force of friction while the body possesses **motion relative to the surface** is known as kinetic friction (f_k), it is always less than limiting static friction (f_s).

$$\text{i.e. } f_k < f_s \quad (\text{where } f_s = f_{s. \text{max}})$$

$$f_k = \mu_k N \quad (\text{where } \mu_k = \text{coefficient of kinetic friction})$$

$$\mu_k < \mu_s$$

Table 3.1 Approximate Coefficients of Friction

Materials	Static, μ_s	Kinetic, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Glass on glass	0.94	0.40
Copper on cast iron	1.05	0.29
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

note For dry, unlubricated surfaces the frictional force follows two laws (1) *It is approximately independent of the area of contact over wide limits*, and (2) *it is proportional to the normal reaction*.

(e) Viscous Forces

These are forces which also oppose or prevent motion. They can be found in **fluids** (i.e gases or liquids)

They are also known as resistance forces and are dissipative in nature. Viscous forces *increase* proportionally with *speed*. If a body (assume

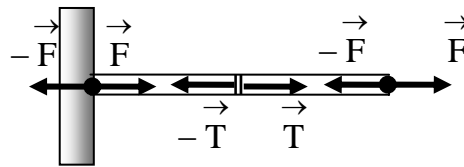
spherical shape having radius, r) possesses *terminal* velocity, v , (maximum constant velocity).

viscous force, $f_v = 6\pi \eta r v$ where, $\eta =$ viscosity.

[It is known as Stokes' Law]

(f) Tension in a String

String is in at rest or moving constant velocity (i.e. $a = 0$ or F_{net} on string = 0)



The string is stretched (or) taut by force F one end and tied to a wall at the other end. The string is now said to be under *tension* and nothing is moving. Since every thing is stationary, the resultant force anywhere and everywhere on the string must be zero. Consider a thin slice of string as shown *equal and opposite forces* must act across the slice since the slice is in equilibrium, so the magnitude of *tensile force* or just *tension* throughout the string is the same of the string is *in equilibrium*.

(g) Tension in a Spring

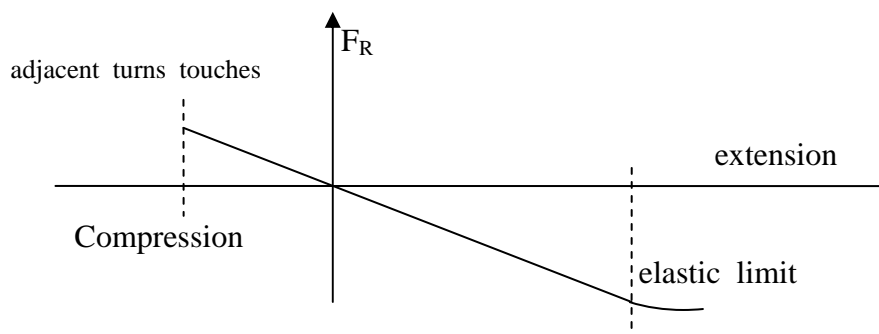
Within the *elastic limit*, restoring force of a spring is directly proportional to the elongation and opposite to the direction of displacement (elongation).

$$F_R \propto -(\Delta \ell)$$

$$F_R = -k \cdot \Delta \ell \quad \text{where } k = \text{spring constant or stiffness}$$

$$k = \left| \frac{F_R}{\Delta \ell} \right|$$

Spring constant is force per unit elongation and its SI unit is N m^{-1} .



Relation between restoring and deformation of a spring

Application of Newton's Laws

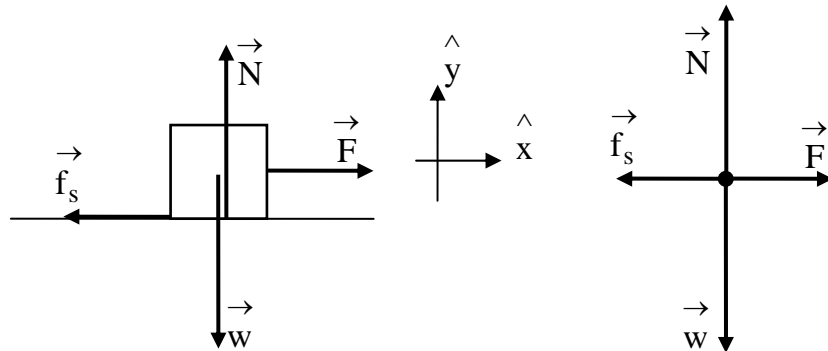
- # Static problem $\Rightarrow a = 0$ [i.e., at rest or moving with constant velocity]
 - must apply Newton's 1st law of motion
- # Dynamic problem $\Rightarrow a \neq 0$
 - must apply Newton's 2nd law of motion

Strategy of Force - Problems

- # Step-1 - Select the body of interest
- # Step-2 - Consider the environment of the body of interest
- # Step-3 - Write down the forces due to environment with the help of suitable (inertial) reference frame.
- # Step-4 - Draw the required free-body diagram [a separated diagram of the interested body alone, showing the reference frame and all the forces acting on the body]
- # Step-5 - Apply Newton's first law, if the problem is 'static'
 Apply Newton's second law if the problem is 'dynamic'

Example 3.1 A block of wood of weight 50 N is resting on horizontal wooden table. A rope is attached to the block and is pulled horizontally. Find (a) the minimum force required to start it moving and (b) the force required to keep it moving with uniform velocity.
 ($\mu_s = 0.4$, $\mu_k = 0.3$)

Solution :



(a) The body of interest is the wooden block. A reference frame is chosen as shown. The forces acting on it are

applied force $\vec{F} = +\hat{x} F$

static friction $\vec{f}_s = -\hat{x} f_s$ (starting condition)

normal reaction $\vec{N} = +\hat{y} N$

weight $\vec{w} = -\hat{y} w$

At the start of motion $F = F_{\min}$
 $f_s = \mu_s N$ (limiting force of static friction)

and $a = 0$

Application of Newton's Law gives

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

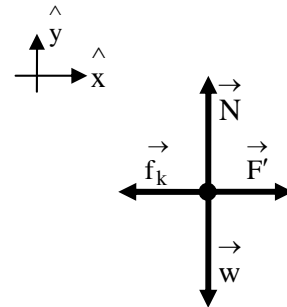
or $F_{\min} - f_s = 0$ $N - w = 0$

or $F_{\min} = \mu_s N$ $N = w$

These two equations together gives

$$F_{\min} = \mu_s w$$

$$= 0.4 \times 50 = 20 \text{ N}$$



(b) For the uniform moving block

$$F = F'$$

and $f_k = \mu_k N$

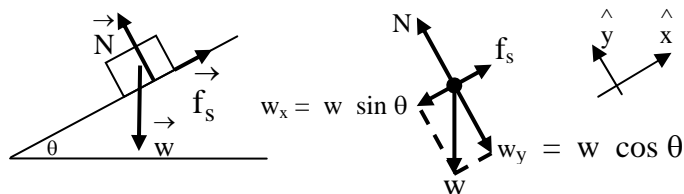
while the other forces are the same .

As before, application of Newton's law gives

$$F' = \mu_k N$$

$$= 0.3 \times 50 = 15 \text{ N}$$

Example 3.2 A block rests on an inclined plane. The angle was adjusted so that it will start sliding down once a slight push is given. Find the angle of inclination of the plane.



The body of interest is the block. A reference frame is chosen as shown. The forces acting on it are

Weight $\vec{w} = -\hat{x} w \sin \theta - \hat{y} w \cos \theta$

Static friction $\vec{f}_s = +\hat{x} f_s$

Normal reaction $\vec{N} = +\hat{y} N$

At the given state of motion

$$f_s = \mu_s N \text{ (limiting force of static friction)}$$

and

$$\vec{a} = 0 \text{ (at rest and remains at rest)}$$

Application of Newton's Law gives

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

$$\text{or} \quad -w \sin \theta + f_s = 0 \quad \text{or} \quad -w \cos \theta + N = 0$$

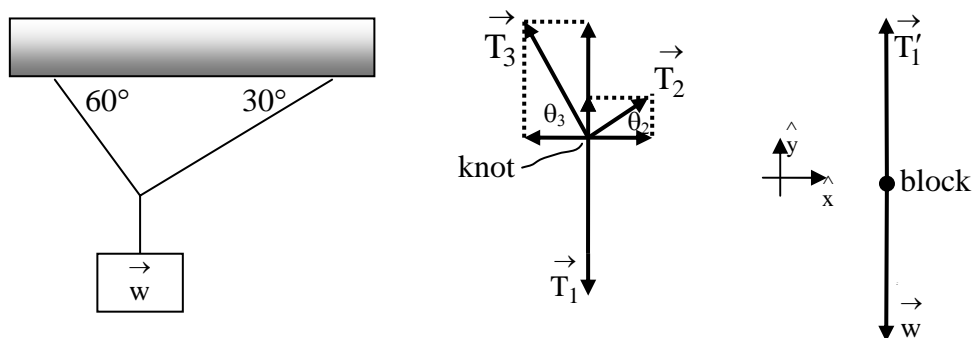
$$\text{or} \quad w \sin \theta = \mu_s N \quad \text{or} \quad w \cos \theta = N$$

These two equations together gives

$$\tan \theta = \mu_s$$

$$\theta = \tan^{-1} \mu_s$$

Example 3.3 A block of weight 200 N is suspended from a ceiling with weightless cords as shown in Fig. Find the tensions in the three cords.



Solution :

A reference frame is chosen as shown.

The first body of interest is the knot. The forces acting on it are,

$$\text{Tension by first cord} \quad \vec{T}_1 = -\hat{y} T_1$$

$$\text{Tension by second cord} \quad \vec{T}_2 = \hat{x} T_2 \cos \theta_2 + \hat{y} T_2 \sin \theta_2$$

$$\text{Tension by third cord} \quad \vec{T}_3 = -\hat{x} T_3 \cos \theta_3 + \hat{y} T_3 \sin \theta_3$$

The condition of motion is $a = 0$

Application of Newton's law gives

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

$$T_2 \cos \theta_2 - T_3 \cos \theta_3 = 0$$

$$T_2 \sin \theta_2 - T_3 \sin \theta_3 - T_1 = 0$$

But, these two equations are not sufficient to solve for the three unknowns, T_1 , T_2 and T_3 .

Consider now the block as the second body of interest. The forces acting on it are,

$$\begin{aligned} \text{Weight} & \quad \vec{w} = -\hat{y} w \\ \text{Tension by first cord} & \quad \vec{T}'_1 = +\hat{y} T' \\ \text{State of motion is} & \quad a = 0 \\ & \quad \Sigma F_y = 0 \\ & \quad T'_1 = w \end{aligned}$$

But T'_1 and T_1 are tensions in the cord under at rest condition, hence they are equal in magnitude.

Substituting the numerical values of $w = 200 \text{ N}$, $\theta_2 = 30^\circ$, $\theta_3 = 60^\circ$, $T'_1 = T_1 = w = 200 \text{ N}$ in respective equations.

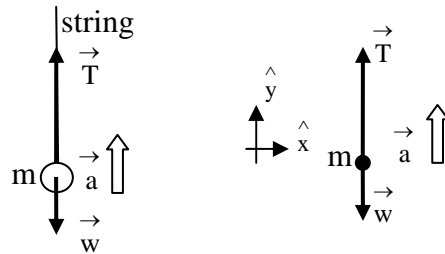
The tensions are

$$T_1 = 200 \text{ N}, \quad T_2 = 100 \text{ N}, \quad T_3 = 173.2 \text{ N}$$

Example 3.4 A particle of mass 50 g is attached to a string and is being pulled upward with an acceleration of 10 cm s^{-2} . What is the upward tension force?

Solution :

A reference frame fixed to the earth is taken as shown. The body of interest is a particle. The forces acting on it are,



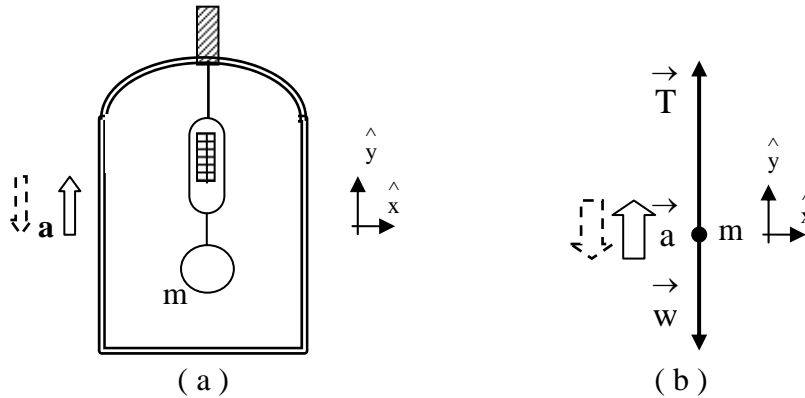
$$\begin{aligned} \text{Weight} & \quad \vec{w} = -\hat{y} w \\ \text{Tension} & \quad \vec{T} = \hat{y} T \\ \text{condition of motion} & \quad a = \hat{y} a \end{aligned}$$

According to the Newton's second law

$$\begin{aligned} \Sigma F_y & = m a_y \\ \text{gives} \quad T - w & = 50 \times 10^{-3} \times 10 \times 10^{-2} \\ T - mg & = 0.005 \\ T & = 0.505 \text{ N} \end{aligned}$$

Example 3.5 A body of mass 'm' is suspended from a spring balance attached to the roof of an elevator, as shown in Fig. What is the reading of the balance if the elevator has an acceleration 'a' relative to the earth? Consider the earth's surface to be an inertial reference system.

Solution



A reference frame is chosen as shown

The balance reads the tension T in the spring. Body of interest is the body of mass m . The forces acting on it are

Weight $\vec{w} = -\hat{j} w$

Tension $\vec{T} = +\hat{j} T$

The motional condition is with constant acceleration, $\vec{a} = \pm \hat{j} a$.

Where, positive sign is to be taken when the acceleration of elevator is upward and negative sign is to be taken when the acceleration of elevator is downward.

Application of Newton's second law

$$\Sigma F_y = m a_y$$

gives $T - w = m (a)$

so, $T = w \pm m a$

Discussion : $T = w$ when ($a = 0$), that is, when the elevator is at rest or is in uniform motion.

$$T = (w + m a) = w'$$

when the elevator accelerates while moving upward or decelerates while moving downward.

$$T = (w - m a) = w''$$

when the elevator accelerates while moving downward or decelerates while moving upward.

$$T = 0$$

When the elevator is freely falling or moving downward with an acceleration $a = g$.

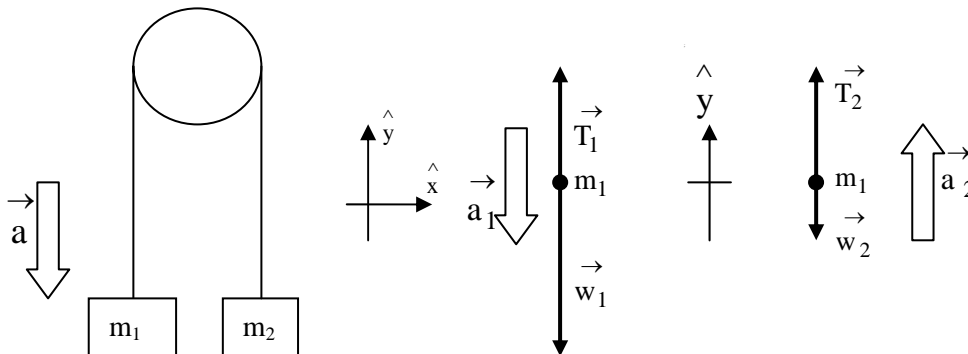
Because we are used to measuring the weight with a spring balance at rest, the tension T is taken commonly as the weight itself.

When the whole system, the balance and the mass is being accelerated we still want to use the word weight for the reading of the balance. But, as $T \neq w$ we call this reading the **apparent weight** of the body.

Example 3.6 Two particles of masses 50 g and 100 g are joined by a string and placed over a frictionless pulley (as in Fig.). This is called an Atwood's machine, and it is assumed that the length of the string does not change and that the masses of the string and pulley are so small that they can be neglected. Find the acceleration of each particle.

Solution I :

The frame of reference is chosen as shown. Our bodies are two particles of mass m_1 and m_2 .



Forces acting on the mass m_1 are:

$$\text{Tension } \vec{T}_1 = +\hat{y} T_1, \quad \text{Weight } \vec{w}_1 = -\hat{y} w_1 = -\hat{y} m_1 g$$

Forces acting on the mass m_2 are:

$$\text{Tension } \vec{T}_2 = +\hat{y} T_2, \quad \text{Weight } \vec{w}_2 = -\hat{y} w_2 = -\hat{y} m_2 g.$$

When the approximation that the pulley used is massless and frictionless, the tensions at the mass-ends of the string are equal (or) $T_1 = T_2 = T$. The string is inextensible which gives us a common acceleration "a" for both m_1 and m_2 but one directed upward and the other downward. Newton's second law,

$$\Sigma F_{1y} = m_1 a_y \quad \text{and} \quad \Sigma F_{2y} = m_2 a_y$$

gives $T - m_1 g = -m_1 a$ $T - m_2 g = m_2 a$

Together they give
$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} \cdot g$$

$$= [(0.1 - 0.05) / (0.1+0.05)] 10 = 3.3 \text{ m s}^{-2}.$$

Solution II :

We may consider the two masses connected to the cord moving together as if it were a single body directed along the motion denoted by unit vector $\hat{\ell}$.

The forces on the system are now

Weight of mass m_1 ,
$$\vec{w}_1 = +\hat{\ell} m_1 g$$

Weight of mass m_2 ,
$$\vec{w}_2 = -\hat{\ell} m_2 g$$

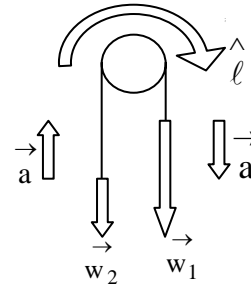
Acceleration of the system
$$\vec{a} = \hat{\ell} a$$

Application of Newton's second law

$$\sum F_{\hat{\ell}} = m a \quad \text{gives} \quad (m_1 - m_2) g = (m_1 + m_2) a$$

$$1-0.5 = (50+100) \times 10^{-3} a$$

$$a = \frac{0.5}{0.15} = \frac{10}{3} = 3.33 \text{ m s}^{-2}.$$



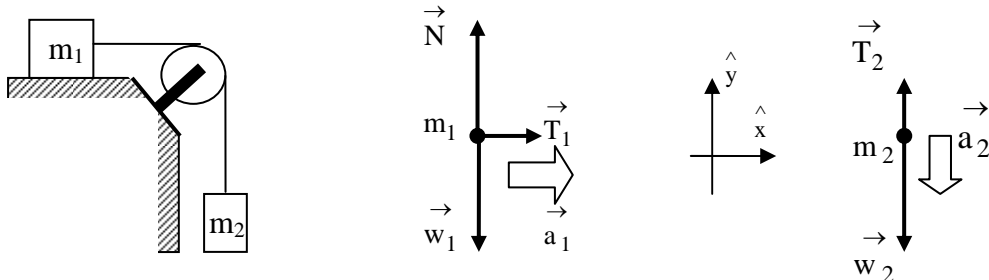
Notice that the magnitude of T is always intermediate between the weight of the mass m_1 (1.0 N in our example) and the weight of the mass m_2 (0.5 N). This is to be expected, since T must exceed $m_2 g$ to give it an upward acceleration, and $m_1 g$ must exceed T to give it a downward acceleration. In the special case when $m_1 = m_2$, we obtain $a = 0$ and $T = m_1 g = m_2 g$, which is the static result we expected.

If we were to drop our assumption of a massless pulley and assign a mass to it, the rotational motion of the pulley results in different tension in each segment of the string. Friction in the bearings also affects the rotational motion of the pulley and the tension in the string.

Example 3.7 Figure shows a block of mass m_1 on a smooth horizontal surface pulley by string which is attached to a block or mass m_2 hanging over a frictionless, massless pulley. Find the acceleration of the system and the tension in string.

Solution

The frame of reference is chosen as shown.



The system of interest is two-body system which consists of block of mass m_1 and block of mass m_2

Forces on block 1 are:

Weight $\vec{w}_1 = -\hat{y} m_1 g$

normal reaction $\vec{N} = \hat{y} N$

tension $\vec{T}_1 = \hat{x} T_1$

State of motion for block 1 $\vec{a}_1 = \hat{x} a_{1x}$

No motion in y-direction as the surface is just supporting the block.

Application of Newton's second law gives,

$$\Sigma F_x = m a_x \quad \text{and} \quad \Sigma F_y = 0$$

yields $T_1 = m_1 a_{1x}$

Forces acting on the block 2 are

Weight $\vec{w}_2 = -\hat{y} m_2 g$

Tension $\vec{T}_2 = +\hat{y} T_2$

State of motion for block 2 $\vec{a}_2 = -\hat{y} a_{2y}$

Then $\Sigma F_y = m_2 a_y$

gives $T_2 - m_2 g = m_2 a_{2y}$

The approximation that the pulley is massless and frictionless, means $T_1 = T_2 = T$.

Since the two bodies in the system move together.

$$a_{2y} = a_{1x} = a$$

where a is the acceleration of the system

We get $m_2 g - T = m_2 a$ and $T = m_1 a$

These yield $m_2 g = (m_1 + m_2) a$

$$a = \frac{m_2}{m_1 + m_2} \cdot g$$

and $T = \frac{m_1 m_2}{m_1 + m_2} \cdot g$

Notice that (1) Tension in the string is always less than $m_2 g$

- (2) 'a' is always less than 'g'
 (3) Only when m_1 equals zero, $a = g$, $T = 0$.

3.3 Dynamic of Circular Motion

To make a circular motion, the *net force* on the body of interest must be a *centripetal* one.

$$\begin{aligned} \text{i.e.} \quad \vec{F}_{\text{net}} &= m \vec{a}_c \\ &= (-\hat{r}) m \frac{v^2}{r} = (-\hat{r}) m r \omega^2 \end{aligned}$$

(where \hat{r} is unit vector that points radially outward.)

$$\omega = \frac{2\pi}{T} \quad \text{and} \quad v = \frac{2\pi r}{T}$$

Example 3.8 Calculate the centripetal force required by a 1500 kg car making a turn of radius 30 m at a speed of 23 km h⁻¹.

Solution :

$$\begin{aligned} \text{The car's mass} \quad m &= 1500 \text{ kg} \\ v &= 23 \text{ km h}^{-1} = 7 \text{ m s}^{-1} \\ \text{Then} \quad F_R &= \frac{m v^2}{r} = \frac{1500 \times 7^2}{30} \\ &= 2450 \text{ N} \end{aligned}$$

The centripetal force of 2450 N must be provided by the pavement acting on the car's tires through the agency of friction. Let us compute the minimum coefficient of friction that must be present if the car is to make the turn without skidding. That frictional force is

$$f_s = \mu_s N$$

in general. To find μ_s , we substitute 2450 N for f_s and 1500×9.8 N for the normal force N , which here is the car's weight "w".

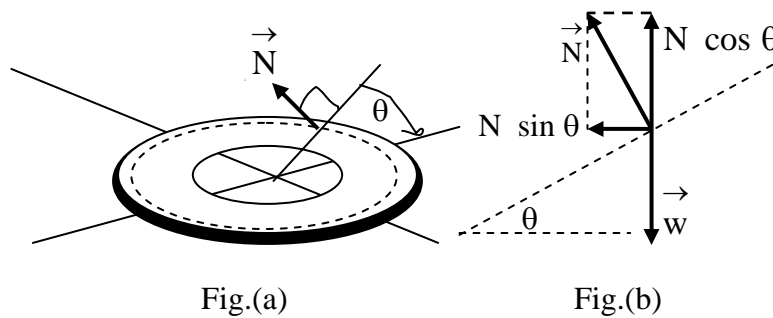
$$\text{Thus} \quad \mu = \frac{f_s}{N} = \frac{F_R}{w} = \frac{2450}{1500 \times 9.8} = 0.17$$

to two significant figures, which readily is available under good driving conditions (Table 3.1).

As seen in the above example, the necessary centripetal force is supplied by the sidewise frictional force exerted by the road on the tires. In the case of a railway car it supplied by the rails exerting a sidewise force on the inner rims of the car's wheels. Neither of these sidewise forces can be safely relied upon to be large enough at all times and both cause

unnecessary wear. Hence the roadbed is banked on curves as shown in the following Fig.(a).

Similarly a turning airplane is supplied with a centripetal force, as a component of aerodynamic (lift) forces created by the flow of air past its wings, when the craft is 'banked' or tilted. If the airplane is tilted to one side, the lift force is still perpendicular to the wings and therefore is tilted as well Fig(b). Only the vertical component of the force supports the plane, and its horizontal component is available as the centripetal force. To make a turn, an airplane simultaneously tilts in the desired direction and moves its rudder to change its heading. If it only does the latter, it will slip sidewise through the air until eventually air such turns would have much larger radii than banked turns.



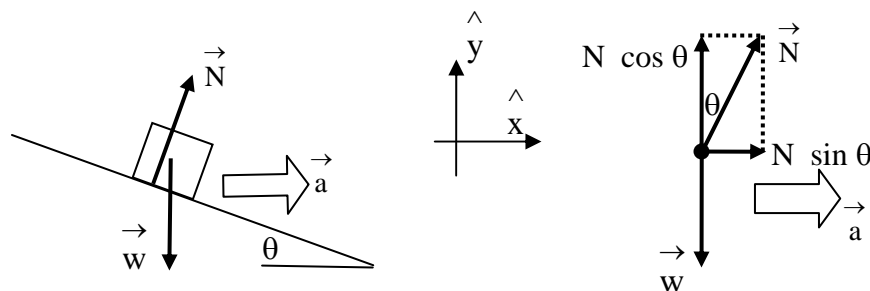
Banking of Curves and Banking angle

If an object is rounding a flat curve, unbanked curve, maximum speed at which that object can move without sliding is controlled by friction.

It is possible to bank curve so that at a certain speed no friction at all is needed. Aeroplanes always bank at this angle when making turns. The angle of such a bank curve is known as **banking angle**.

Example 3.9 Find the banking angle of the roadbed or of the airplane wings so as to enable the automobile, the railway car, or the aircraft to turn in a circle of radius R at a speed v without taking the help of friction or the sidewise force of the rails.

A reference frame is chosen as shown



We will consider all the three cases together. Only two forces act on each body, they are

$$\text{Weight} \quad \vec{w} = -\hat{y} \, m g$$

$$\text{Normal reaction} \quad \vec{N} = \hat{y} \, N \cos \theta + \hat{x} \, N \sin \theta$$

(\vec{N} is the reaction force of the roadbed on the automobile, the reaction of the rails on the rail-car or the aerodynamic lift of the air on the airplane)

$$\text{Condition of motion} \quad \vec{a} = \hat{x} \, a_R = \hat{x} \, \frac{v^2}{R}$$

(The body requires the centripetal acceleration v^2/R to turn on its own.)

Application of Newton's law gives

$$\Sigma F_x = m a_x \quad \text{and} \quad \Sigma F_y = 0 \quad (\text{No motion in y-direction})$$

$$\text{gives} \quad N \sin \theta = \frac{m v^2}{R} \quad \text{and} \quad N \cos \theta = m g$$

$$\text{We get} \quad \tan \theta = \frac{v^2}{R g} .$$

Notice that the proper angle of banking depends upon the speed and the curvature of the path. For a given curvature, the road is banked at an angle corresponding to an expected average speed. Often curves are marked by signs giving the proper speed for which the road was banked.

The student should check the banking formula for limiting cases of $v = 0$; $R = \infty$; v large ; and R small.

fsm01vd1

fsm02vd2

fsm03vd1

fsm03vd2

fsm03vd3

fsm03vda

fsm04vd1

fsa05vd1

fsa05vd2

fsa06an1

fsa07an1

fsb06vd1

fsbo6vd2

fsc06vd1

fsm07vd1

fsm08vd2

fsm09vd1

fsx04vd1

fsx08vd1

3.4 Impulse

The impulse of a force, in general, is the integral of a force over the time interval during which the force acts.

Mathematically

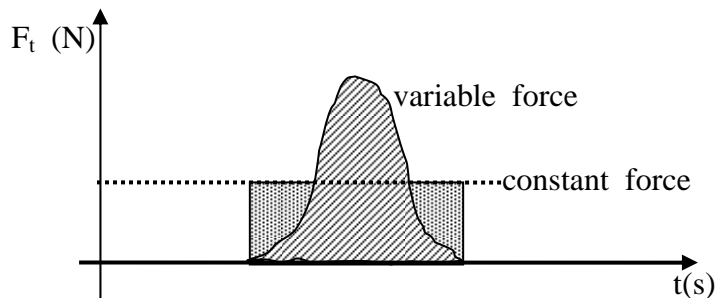
$$\begin{aligned} \text{Impulse} &= \int_{t_i}^{t_f} F dt \\ &= \text{area under } F - t \text{ graph} \end{aligned}$$

For a constant force,

$$\text{impulse} = F \cdot \Delta t$$

i.e., the product of the constant force and the time interval during which the force acts.

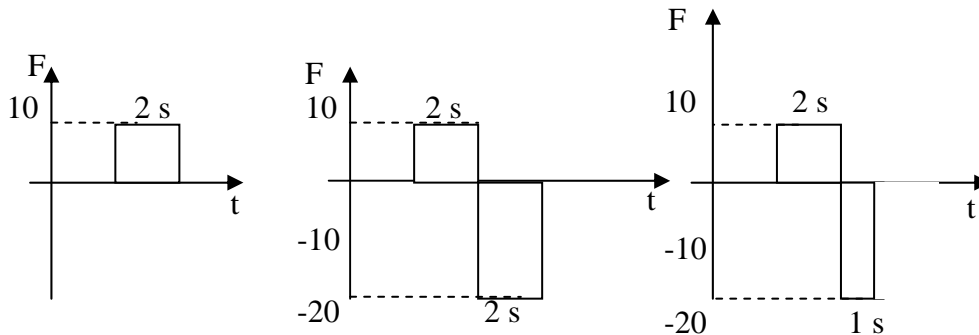
SI unit of impulse is (**N.s**). It is a **vector** quantity.



Example 3.10 Consider the changes in momentum produced by the following forces (a) A body moving on the x-axis is acted on for 2 s by a constant force of 10 N toward the right. (b) The body is acted on for 2 s by a constant force of 10 N toward the right and then for 2 s by a constant force of 20 N toward the left (c) The body is acted on for 2 s by a constant force of 10 N toward the right, then for 1 s by a constant force of 20 N toward the left. The three forces are shown graphically in the following figure.

Solution :

(a) The impulse of the force is $+10 \times 2 = +20 \text{ N s}$. Hence the



momentum of any body on which the force acts increase by 20 kg m s^{-1} . This change is the same whatever the mass of the body and whatever the magnitude and direction of its initial velocity.

Suppose the mass of the body is 2 kg and that initially it is at rest. Its final momentum then equals its changes in momentum and its final velocity is 10 m s^{-1} toward the right. (The reader should verify by computing the acceleration)

$$a = (v - v_0) / t = (10 - 0) / 2 = 5 \text{ m s}^{-2}$$

$$F = m a = 2 \times 5 = 10 \text{ N}$$

$$F.t = 10 \times 2 = 20 \text{ N s}$$

$$\text{change in momentum} = 20 \text{ kg m s}^{-1}$$

Had the body been initially moving toward the **right** at 5 m s^{-1} , its initially momentum would have been 10 kg m s^{-1} , its final momentum 30 kg m s^{-1} , and its final velocity 15 m s^{-1} toward the **right**.

Had the body been initially moving toward the **left** at 5 m s^{-1} , its initial momentum would have been -10 kg m s^{-1} , its final momentum $+10 \text{ kg m s}^{-1}$, and its final velocity 5 m s^{-1} toward the **right**. That is, the constant force of 10 N toward the right would first have brought the body to rest and then give it a velocity in the direction opposite to its initial velocity.

(b) The impulse of this force is $(+10 \times 2 - 20 \times 2) = -20 \text{ N s}$. The momentum of any body on which it acts is decreased by 20 kg m s^{-1} . The reader should examine various possibilities, as in the preceding example.

(c) The impulse of this force is $(+10 \times 2 - 20 \times 1) = 0$. Hence the momentum of any body on which it acts is not changed. Of course, the momentum of the body is increased during the first 2 s but is decreased by an equal amount in the next second. As an exercise, describe the motion of the body of mass 2 kg moving initially to the **left** at 5 m s^{-1} , and acted by this force. It will help to construct a graph of velocity versus time.

Example 3.11 A ball of mass 0.4 kg is thrown against a brick wall. When it strikes the wall it is moving horizontally to the left at 30 m s^{-1} , and it rebounds horizontally to right at 20 m s^{-1} . Find the impulse of the force exerted on the ball by the wall.

Solution :

The initial momentum of the ball is $0.4 \times (-30) = -12 \text{ kg m s}^{-1}$

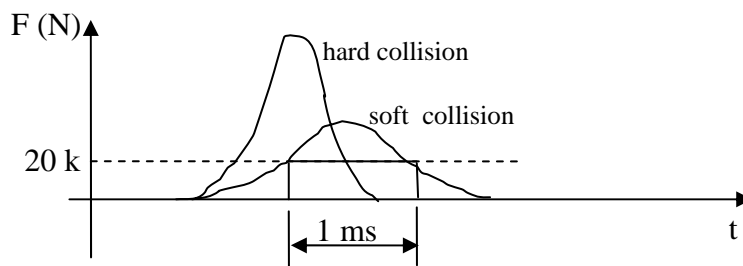
The final momentum is $+8 \text{ kg m s}^{-1}$, The change in momentum is $m v_2 - m v_1 = +8 - (-12) = 20 \text{ kg m s}^{-1}$.

Hence, the impulse of the force exerted on the ball was 20 N s . Since the impulse is **positive**, the force must be toward the **right**.

Note that the force exerted on the ball cannot be found without further information regarding the collision. The general nature of the force-time graph is shown by one of the graphs as in the following Figure. The force is zero before impact, rises to a maximum and decreases to zero when the ball leaves the wall.

If the ball is relatively rigid like a baseball the time of collision is small and the maximum force is large, as in curve (a). If the ball is more yielding like a tennis ball, the collision time is larger and the maximum force is less, as in curve (b). In any event, the area under the force-time graph must equal 20 N s. For an idealized case in which the force is a constant and the collision time is 1 ms (10^{-3} s), as represented by the horizontal straight line, the force is 20,000 N.

Explain why one “eases off” when catching a fast ball.



3.5 Linear Momentum : Linear Momentum (\vec{p}) of a particle is product of its mass (m) and velocity (\vec{v}).

It is a **vector** quantity and its SI unit is **kg m s⁻¹**.

Mathematically $\vec{p} = m \vec{v}$

3.6 Impulse - Momentum Theorem

From the Newton's second law of motion,

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \quad (\text{or}) \quad \vec{F}_{\text{net}} = m \vec{a} = \frac{m(\vec{v} - \vec{v}_0)}{t} = \frac{m\vec{v} - m\vec{v}_0}{t}$$

$$\therefore \vec{F} dt = d\vec{p}$$

$$\int_{t_i}^{t_f} \vec{F} dt = \int_{p_i}^{p_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta \vec{p}$$

$$\int \vec{F} dt = \Delta \vec{p}$$

impulse = change in momentum

This is called the **impulse-momentum theorem**.

3.7 Principle of conservation linear of momentum

The total momentum of a system is constant if **no** external resultant force acts on it. (or) The total momentum of an **isolated system** is constant

In symbol, **If**

$$\vec{F}_{\text{net}} = 0 \quad \text{then} \quad \sum \vec{p} = \text{constant} \quad (\text{or}) \quad \Delta \vec{p} = \text{constant}$$

Note: The principle of conservation of momentum is actually an extension of Newton's second law and third laws of motion. It is nothing new.

$$\text{i.e. If } \vec{F}_{\text{net}} = 0, \quad \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = 0 \quad \text{and} \quad \vec{p} = \text{constant.}$$

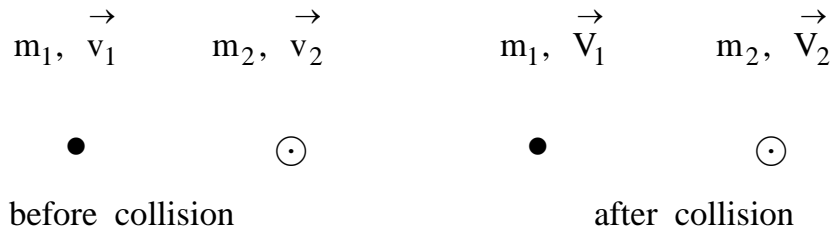
If bodies collide, at the contact point a pair of action - reaction forces is formed. Since they are equal and opposite, the resultant **external** force acting on the system is therefore zero.

This is the required condition for the *principle of conservation of linear momentum*.

3.8 Simple Collisions

- ⇒ An *elastic collision* (perfectly elastic collision) is a collision in which the **kinetic energy** of the system is *conserved*.
- ⇒ A *perfectly inelastic collision* in a collision in which *not only* is the kinetic energy of the system **not conserved**, but the two colliding bodies *also* move as *one* (i.e. **combined into one**), after collision.
- ⇒ The principle of conservation of linear momentum is applicable to both types of collisions mentioned above.
- ⇒ The loss of kinetic energy for the case of perfectly inelastic collisions appears as increased internal energy, which is eventually dissipated as heat loss, deformation (change of potential energy), and to a lesser extent in the form of sound energy.

3.8.1 Elastic collision (perfectly elastic collision)

**Law of conservation of momentum**

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{V}_1 + m_2 \vec{V}_2$$

in component form [for 2 - D]

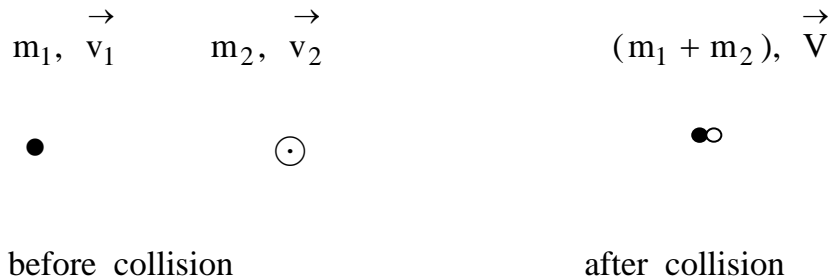
$$m_1 v_{1x} + m_2 v_{2x} = m_1 V_{1x} + m_2 V_{2x}$$

$$m_1 v_{1y} + m_2 v_{2y} = m_1 V_{1y} + m_2 V_{2y}$$

(can also be used with signed scalar form)

Conservation of kinetic energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

3.8.2 Perfectly inelastic collision**Law of conservation of momentum**

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{V}$$

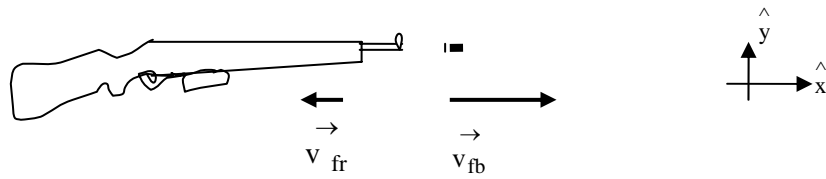
in component form, [for 2 - D]

$$m_1 v_{1x} + m_2 v_{2x} = (m_1 + m_2) V_x$$

$$m_1 v_{1y} + m_2 v_{2y} = (m_1 + m_2) V_y$$

Those two equations can also be used with signed scalar form.

Example 3.12 A rifle of mass 2 kg fires a bullet of mass 10 g at a muzzle speed of 600 m s^{-1} (as in following figure.). What is the recoil speed of the rifle?



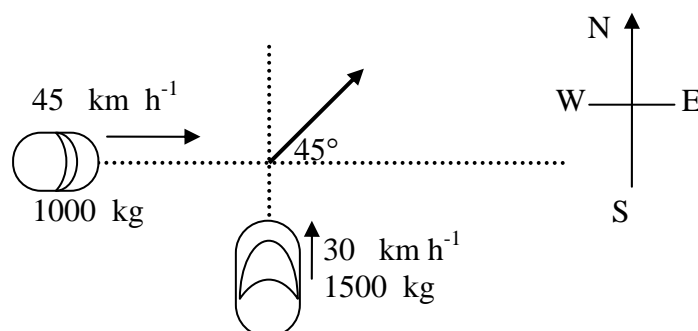
Solution :

Mass of rifle	$m_r = 2 \text{ kg}$
Mass of the bullet	$m_b = 10 \text{ g} = 10^{-2} \text{ kg}$
Initial velocity of the rifle	$\vec{v}_{ir} = 0$
Initial velocity of the bullet	$\vec{v}_{ib} = 0$
Final velocity of the rifle	$\vec{v}_{fr} = ?$
Final velocity of the bullet	$\vec{v}_{fb} = \hat{x} 600 \text{ m s}^{-1}$
Initial total momentum of the system	$= m_r \vec{v}_{ir} + m_b \vec{v}_{ib}$
Final total momentum of the system	$= m_r \vec{v}_{fr} + m_b \vec{v}_{fb}$

According to the principle of conservation of momentum

$$\begin{aligned}
 m_r \vec{v}_{ir} + m_b \vec{v}_{ib} &= m_r \vec{v}_{fr} + m_b \vec{v}_{fb} \\
 0 &= 2 \vec{v}_{fr} + 10^{-2} (600) \\
 \vec{v}_{fr} &= -3 \text{ m s}^{-1} \\
 \vec{v}_{fr} &= -\hat{x} 3 \text{ m s}^{-1}.
 \end{aligned}$$

Example 3.13 Two cars travelling, one from west to east and the other from south to north, collide. They got stuck and move together after collision. The initial speed of the first one is 45 km h^{-1} and that of the second is 30 km h^{-1} . The masses of the first and second cars are 1000 kg and 1500 kg respectively, What is the velocity of the wreck after collision?



Solution : We will call the W-E direction, the positive x-axis and that of S-N as y-axis.. The final mass is 2500 kg. Momentum is conserved: which implies that the x-and y-components of the total momentum are separately conserved. Thus, in units convenient for the problem though not, strictly speaking, proper momentum units.

$$\text{x- component} \quad 1000 \times 45 = 2500 v_x \text{ kg km h}^{-1}$$

$$\text{y- component} \quad 1500 \times 30 = 2500 v_y \text{ kg km h}^{-1}$$

$$\text{Therefore} \quad v_x = 18 \text{ km h}^{-1} \quad \text{and} \quad v_y = 18 \text{ km h}^{-1}$$

Where v_x and v_y are x-and y-components of the velocity of the wreck.

The magnitude of the final velocity is

$$v = \sqrt{(v_x^2 + v_y^2)} = 25.5 \text{ km h}^{-1}.$$

The direction is along a line making an angle of $\tan^{-1} \left(\frac{v_y}{v_x} \right) = 45^\circ$ with the x-axis, which is north-east.

3.9 Rotational Motion

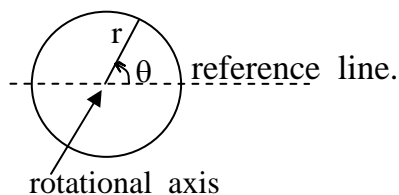
A **rigid body** is a body with a **definite** and **unchanging shape and size**

When a rigid body *rotates* about a **stationary axis**, its motion is described by its **angular position**, **angular velocity** and **angular acceleration**.

3.9.1 Angular Position

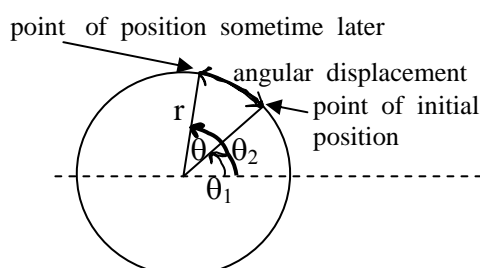
Measured angle is counter **clockwise** direction from a reference line.

Its unit is '**radian**'



3.9.2 Angular Displacement:

Change in angular position



$$\Delta \theta = \theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

Its unit is also '**radian**'

Note: If $s = r$, $\theta = (s/r) = 1$ radian $\approx 57.3^\circ$

3.9.3 Average angular velocity

$$\text{average angular velocity} = \frac{\text{total angular displacement}}{\text{total time taken}}$$

$$\omega_{\text{av}} = \frac{\Delta \theta}{\Delta t}$$

3.9.4 Instantaneous angular velocity

It is defined as the rate of change of angular displacement (with time).

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Its SI unit is '**rad s⁻¹**'

[Other common unit : revolution per minute (r.p.m)]

$$1 \text{ rpm} = \frac{2\pi}{60} \text{ rad s}^{-1}$$

Angular velocity is related to the '**frequency**' of rotation.

If '**n**' revolutions are made in one second, then $\omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi n}{1}$ gives,

$$\omega = 2\pi n$$

3.9.4 Average angular acceleration

$$\alpha_{\text{av}} = \frac{\text{change in angular velocity}}{\text{total time taken}}$$

$$\alpha_{\text{av}} = \frac{\Delta \omega}{\Delta t}$$

3.9.6 Instantaneous angular acceleration